A multi-item lot-sizing problem with bounded number of setups

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Keywords: *Lot-sizing, Mixed Integer Programming, Simulation, Stochastic Optimization*

1 Context

Companies live currently in an environment that is always more competitive and changing. At a strategic level, they face contradictory challenges: reduction of costs and increase of *flexibility* (capability to change the composition of their product mix). How to deal with these opposite objectives? This is the main theme of a CIFRE thesis produced through collaboration between Argon Consulting and Ecole des Ponts ParisTech. A first problem met during this work is tactical and takes the form of a multi-item lot-sizing problem, which to the best of our knowledge has not yet been studied in academics. Inventory is the unique source of costs. There is a weekly demand over a finite horizon for various references produced by an assembly line, with an upper bound on the week production and, this seems to be quite original, with a "setup upper bound" on the maximal number of distinct references produced per week. A similar bound has been considered by Rubaszewski *et al.* [3]. Contrary to what holds for our problem, their bound is an overall bound for the production over the whole horizon, and they still consider setup costs. According to the experience of Argon Consulting experts, while inventory costs and this setup bound are easy to estimate, setup costs are hard to quantify. The purpose of the present work is to study this problem, as well as a stochastic extension, which is more relevant in practice. For a bibliographical account, see [1] and [2], which provide surveys respectively for deterministic and stochastic lot-sizing problems.

2 Deterministic case

The assembly line produces a set $\mathcal{R}$ of references over T weeks. The maximum number of distinct references that can be produced over a week is N. We denote by d_t^r the demand for reference r over the week t and by h^r the inventory cost for reference r over one week. The parameter γ is a penalization for *backorders*, i.e., negative inventory levels. The multi-item lot-sizing problem with bounded number of setups we address in the present work can be modeled as the mixed integer program (P) below, with the following variables: the variable $\tilde{s}_t^r$ (resp. b_r^t), which models the inventory (resp. the backorder) of reference r at the end of week t, the variable q_t^r, which models the quantity of reference r produced over week t, and the variable x_t^r, which takes the value 1 if the reference r is produced over week t, and is allowed to take the value 0 otherwise. Notice that the quantities are normalized in the model.

Problem (P) is $\mathcal{NP}$-hard in the strong sense for any fixed $N \geq 3$, as shown by a straightforward reduction of 3-PARTITION to our problem. We have not been able to settle the case when $N = 1$ or $N = 2$ yet. Another aspect of the hardness of the problem is captured by the weakness of its continuous relaxation: the value of the continuous relaxation of (P) is independent of N. We also prove that some natural extended formulations have the same optimal value as the continuous relaxation.

$$
\begin{array}{lll}
\min & \displaystyle\sum_{t=1}^{T}\sum_{r\in\mathcal{R}}(h^r\tilde{s}_t^r+\gamma b_t^r) & \\
\text{s.t.} & s_t^r=\tilde{s}_t^r-b_t^r & \forall t\in[T],\forall r\in\mathcal{R}\\
& s_t^r=s_{t-1}^r+q_t^r-d_t^r & \forall t\in[T],\forall r\in\mathcal{R}\\
& \displaystyle\sum_{r\in\mathcal{R}}q_t^r\le 1 & \forall t\in[T]\\
& q_t^r\le x_t^r & \forall t\in[T],\ \forall r\in\mathcal{R}\\
& \displaystyle\sum_{r\in\mathcal{R}}x_t^r\le N & \forall t\in[T]\\
& x_t^r\in\{0,1\} & \forall t\in[T],\ \forall r\in\mathcal{R}\\
& q_t^r,\ \tilde{s}_t^r,\ b_t^r\ge 0 & \forall t\in[T],\ \forall r\in\mathcal{R}.
\end{array}
\tag{P}
$$

3 Stochastic case and experiments

The stochastic version of the lot-sizing problem dealt with in this work uses the same parameters as for the deterministic problem described in Section 2, except that the demand is now random with given probability law. At the beginning of each week, the references and the quantities produced have to be decided for that week, but it is not required to determine them for the next weeks. A mathematical program of the problem takes the following form.

$$
\begin{array}{lll}
\min & \mathbb{E}\left[\displaystyle\sum_{t=1}^{T}\sum_{r\in\mathcal{R}}(h^r\tilde{\boldsymbol{s}}_t^r+\gamma \boldsymbol{b}_t^r)\right] & \\
\text{s.t.} & \boldsymbol{s}_t^r=\tilde{\boldsymbol{s}}_t^r-\boldsymbol{b}_t^r & \forall t\in[T],\ \forall r\in\mathcal{R}\\
& \boldsymbol{s}_t^r=\boldsymbol{s}_{t-1}^r+\boldsymbol{q}_t^r-\boldsymbol{d}_t^r & \forall t\in[T],\ \forall r\in\mathcal{R}\\
& \displaystyle\sum_{r\in\mathcal{R}}\boldsymbol{q}_t^r\le 1 & \forall t\in[T]\\
& \boldsymbol{q}_t^r\le \boldsymbol{x}_t^r & \forall t\in[T],\ \forall r\in\mathcal{R}\\
& \displaystyle\sum_{r\in\mathcal{R}}\boldsymbol{x}_t^r\le N & \forall t\in[T]\\
& \boldsymbol{x}_t^r\in\{0,1\} & \forall t\in[T],\ \forall r\in\mathcal{R}\\
& \boldsymbol{q}_t^r,\ \tilde{\boldsymbol{s}}_t^r,\ \boldsymbol{b}_t^r\ge 0 & \forall t\in[T],\ \forall r\in\mathcal{R}\\
& \sigma(\boldsymbol{q}_t^r),\ \sigma(\boldsymbol{x}_t^r)\subset\sigma\big((\boldsymbol{d}_0^r,\dots,\boldsymbol{d}_t^r)_{r\in\mathcal{R}}\big) & \forall t\in[T],\ \forall r\in\mathcal{R}.
\end{array}
\tag{S}
$$

The variables have the same meaning as their deterministic counterparts of Section 2, except that they are now random. The last constraint of the program imposes that the decisions are measurable with respect to what is known when these decisions have to be taken (the planner does not know the future). For $t=1$, the decisions depend only on the realizations of the $\boldsymbol{d}_0^r$'s.

Problem (S) is hard to solve. We are currently performing simulations on real data to compare three approaches, all of them by recomputing each week a solution taking into account the current inventory and the updated forecast: the heuristic used by Argon Consulting, the solution of the deterministic program (P), and a two-stage stochastic approximation of (S).

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