Vehicle Routing Problems on Road Networks

Hamza Ben Ticha[1], Nabil Absi[1], Dominique Feillet[1], Alain Quilliot[2]

[1] Ecole des Mines de Saint-Etienne and UMR CNRS 6158 LIMOS CMP F-13541 Gardanne, France
{hamza.ben-ticha,absi,feillet}@emse.fr

[2] LIMOS Institut Supérieur d'Informatique de Modélisation et leurs Applications, Aubière, France
{alain.quilliot}@isima.fr

Mots-clï¿½s : *VRP, road network, multigraph, branch-and-price, ALNS.*

1 Introduction

Vehicle routing problems have drawn researchers' attention for more than fifty years. Most approaches found in the literature are based on the key assumption that for each pair of points of interest (e.g., customers, depot...), the best origin-destination path can be computed. Then, the problem can be addressed via a simple graph representation, where nodes represent points of interest and arcs represent these best paths. In practice, it is common that several attributes are defined on road segments. Consequently, alternative paths presenting different trade-offs exist between points of interest. In such situations, representing the problem with a simple graph could discard many potentially good solutions from the solution space [2].

A typical example is provided by the Vehicle Routing Problem with Time Windows (VRPTW). In this problem, transportation plans are constrained to satisfy customer requests within their time windows. Each road segment is defined with a cost and a travelling time. Typically, the problem is defined on a simple graph and each arc represents a best path. However, the cheapest path is unlikely the same as the fastest path. When defining its transportation plan, a carrier might prefer an expensive road segment in case of hard time constraints or, conversely, a cheapest one when time constraints are soft. Consequently, representing the problem with a simple graph could lead to operational solutions with an overestimated cost, or, even worse, to the false conclusion that no feasible solution exists.

In this study, we investigate in depth the limits of the simple graph representation of the road network. For this, we propose first to tackle the VRPTW with a multigraph representation that permits to maintain all these alternative paths in the solution space. Then, we propose to solve the problem directly on the road network.

2 VRPTW with a multigraph representation

We propose to represent the road network with a multigraph, so that an arc is introduced for a non-dominated path linking two points of interest in the road network. We develop a Branch-and-Price algorithm and a heuristic method.

2.1 Branch-and-Price algorithm

We develop a Branch-and-Price procedure for the multigraph based VRPTW. The master problem has a similar form as for the standard VRPTW. The pricing problem can be reduced to a multigraph-based Elementary Shortest Path Problem with Resource Constraints (MG-ESPPRC). We adapt the dynamic programming algorithm proposed in [1] to handle parallel arcs between each pair of nodes: labels at a node i are extended on all outgoing arcs $(i,j)^p$. As branching rule, we propose to select an arc $(i,j)^p$ with a fractional flow and then to derive two branches; in the first branch we enforce the use of the arc $(i,j)^p$ so that all considered routes where customer j is served immediately after customer i use arc $(i,j)^p$ and in the second branch, we forbid the arc $(i,j)^p$, for this we remove all routes using $(i,j)^p$ and remove arc $(i,j)^p$ from the considered multigraph in the pricing phase.

2.2 Heuristic method

We propose a heuristic method based on an Adaptive Large Neighborhood Search which consists in the destruction and the repair of a solution to explore different promising area in the solution space. An adapted Savings algorithm is used to construct an initial solution. In the destruction phase, three removal heuristics are used. Three insertion heuristics are used in the repair phase. These heuristics are used alternatively: one removal heuristic and one insertion heuristic are selected based on statistics obtained during the search. Note that, with the multigraph representation, even elementary operations (removal or insertion of a node) become difficult to evaluate and induce an NP-hard problem. This is because the sequence of arcs used to link customers along a route must be re-optimized at each operation. To handle this issue, we propose a procedure based on an incremental data structure and dynamic programming algorithm.

3 VRPTW on road network

In a second part of this study, we propose to tackle the VRPTW directly on the original road network. We propose a complete Branch-and-price scheme based the road network. For this, we denote by $G=(V,A)$ the considered network where V represents the set of nodes (depot, customer locations and road junctions) and A represents the set of road segments. We denote by 0 the depot node and by $C \subseteq V/\{0\}$ the set of customer locations. The pricing problem for the VRPTW on road network is solved using pricing routines similar to those presented by Letchford et al. [3] and based on a dynamic programming approach. The branching scheme used for this setting consists in selecting an arc (i,j) with a fractional flow $\bar{\phi_{ij}}$ and branching on this arc flow. Then, two branches are generated: in the first branch, we ensure that flow on arc (i,j) $\phi_{ij} \leq \lfloor \bar{\phi_{ij}} \rfloor$ and in the second branch, we ensure that $\phi_{ij} \geq \lfloor \bar{\phi_{ij}} \rfloor + 1$.

4 Computational experiments

Experiments are carried out on modified benchmarks from literature and on instances derived from real data. The obtained results show that using the proposed approaches, solution cost is improved for 83 and 136 out of 177 instances compared to solutions obtained on, respectively, simple graph with less costly arcs (min-cost graph) and simple graph with fastest paths (min-time graph). Costs are reduced up to 13.1% against costs on min-cost graphs and up to 53.8% against costs on min-time graphs.

References

[1] Dominique Feillet, Pierre Dejax, Michel Gendreau, and Cyrille Gueguen. An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems. *Networks*, 44(3):216–229, 2004.

[2] Thierry Garaix, Christian Artigues, Dominique Feillet, and Didier Josselin. Vehicle routing problems with alternative paths: An application to on-demand transportation. *European Journal of Operational Research*, 204(1):62–75, 2010.

[3] Adam N Letchford, Saeideh D Nasiri, and Amar Oukil. Pricing routines for vehicle routing with time windows on road networks. *Computers & Operations Research*, 51:331–337, 2014.