The stochastic berth allocation problem

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1 Introduction

The BAP (Berth Allocation Problem) that we study consists in allocating berthing positions on a quay and mooring times to incoming ships in a container terminal, with the objective to minimize their turnaround times, that is, the total time they will spend at the port. The BAP, which is a $\mathcal{NP}$-hard problem, has received significant attention in the literature in operational research (see the survey [1]), but only a dozen of the hundred of studies consider explicitly the uncertainty on the arrival times of ships, for example [3]. However, in 2007, a large survey revealed that over 40% of the container ships deployed on worldwide liner services arrive one or more days behind the initial schedule [4].

Therefore, we extend the discrete model of the berth allocation problem, well established in the literature, to the case of stochastic arrival times. In this model, the quay is partitioned into segments, referred to as berths, which can each accommodate one ship at a time. The service duration of a ship depends on the berth which it is assigned to. The deterministic discrete berth allocation problem corresponds to the scheduling problem $R \mid r_i \mid \sum C_i$, with additional start and end of availability for each machine, consisting in scheduling tasks (ships) with ready times on unrelated parallel machines (berths) to minimize total completion time. In the stochastic setting, we consider two lexicographic objectives: first, to maximize the probability of feasibility, regarding the ends of availability of the berths, second, to minimize the expected total turnaround time (the expected total flow time, that is, $\mathrm{E}[\sum_{i=1}^{n}(C_i - r_i)]$).

The solution approach that we propose to solve the stochastic berth allocation problem combines a proactive and a reactive phase. In the proactive phase, a planning is computed: a berth assignment to ships and a sequencing of ships in berths. In the reactive phase, the berth assignment to ships is fixed and ships are dynamically re-sequenced in berths as they arrive. Thus, we offer the following novel contributions:

- an efficient algorithm that allows to evaluate exactly each planning, incorporated into an iterated tabu search heuristic,
- an exact stochastic dynamic programming algorithm for the optimal dynamic management of each berth,
- a global decision support system that combines the two previous contributions,
- numerical results that provide insights about the value of stochastic information for the BAP.

2 Proactive phase: berth assignment, ship sequencing

It is assumed that the random variables describing the arrival times of ships are discrete and independently distributed. It is also assumed that all time measurements are integer. Data are introduced in table 1.

TAB. 1: Data of the stochastic berth allocation problem

B	set of berths
V	set of ships
$[\![s_b, e_b]\!]$	availability of berth $b \in B$
p_v^b	service duration of ship v in berth b, $v \in V$, $b \in B$
$[\![r_v^{\min}, r_v^{\max}]\!]$	possible arrival times t of ship $v \in V$
$\Pr(r_v = t)$	probability that ship $v \in V$ arrives at time $t \in [\![r_v^{\min}, r_v^{\max}]\!]$

We first introduce an algorithm that, given a planning - a berth assignment to ships and a sequencing of ships in berths - computes in $\mathcal{O}(V\tau)$ the probability distributions of the completion times of ships C_v, with $\tau = \max\{r_v^{\max} - r_v^{\min} \mid v \in V\}$. This algorithm is very useful, since it allows to evaluate exactly the probability of feasibility and the expected turnaround time of a planning during the execution of a solution method. It is based on the following forward-recurrence equations. For the first ship in the sequence in berth b:

$$\Pr(C_1 = t + p_v^b) = \Pr(r_v \leq s_b), \quad \text{if } t = s_b, \tag{1}$$

$$\Pr(C_1 = t + p_v^b) = \Pr(r_v = t), \quad \text{if } t > s_b. \tag{2}$$

For the ships 2 to σ_b in the sequence in berth b:

$$\Pr(C_v = t + p_v^b) = \Pr(r_v \leq t-1)\Pr(C_{v-1} = t) + \Pr(r_v = t)\Pr(C_{v-1} \leq t) \qquad \forall t \in [\![t_v^{\min}, t_v^{\max}]\!],$$

where $t_v^{\min}$ and $t_v^{\max}$ are respectively the minimum and maximum times at which the service of ship v may start, with $t_v^{\min} = \max\{r_v^{\min}, C_{v-1}^{\min}\}$ and $t_v^{\max} = \max\{r_v^{\max}, C_{v-1}^{\max}\}$. The first term of equation (3), $\Pr(r_v \leq t-1)\Pr(C_{v-1} = t)$, corresponds to the case where ship v arrives before the completion of ship $v-1$ and has to wait. The second term, $\Pr(r_v = t)\Pr(C_{v-1} \leq t)$, corresponds to the case where ship v arrives after the completion of ship $v-1$ and receives immediate service.

For the value of the first objective, the probability of feasibility is:

$$\prod_{b \in B} \sum_{\substack{t \in [\![C_{v_b}^{\min}, C_{v_b}^{\max}]\!]: \\ t \leq e_b}} \Pr(C_{v_b} = t),$$

where v_b denotes the last ship in the sequence of berth b.

For the value of the second objective, the expected total turnaround time is:

$$\sum_{v \in V} \sum_{t = C_v^{\min}}^{C_v^{\max}} \Pr(C_v = t)t - \sum_{v \in V} \overline{r}_v,$$

where $\overline{r}_v$ is the average arrival time of ship v.

We address the solution of the stochastic planning problem with a heuristic approach, based on the T^2S heuristic introduced by [2]. The T^2S heuristic is a tabu search, which we extend with the evaluation algorithm briefly described above, and which we integrate into an iterated tabu search, that provides almost optimal solutions.

3 Reactive phase: re-sequencing

Here, the dynamic management of a single berth is considered, to which a set of ships have been assigned. Note that, when the constraint on the end of availability of the berth is relaxed, and for all $v \in V$, $r_v^{\min} = r_v^{\max}$, the problem amounts exactly to schedule tasks on one machine with release dates while minimizing total completion time, which is $\mathcal{NP}$-hard. Hence, the more general problem of the dynamic management of one berth is $\mathcal{NP}$-hard.

A decision has to be taken at the beginning of each period, when the berth is idle and at least one ship is waiting. We associate with any of these times, called decision times, a state defined as a triplet (t, V_B, V_S), where t is the current time, V_B is the set of waiting ships, and V_S the set of incoming ships.

The evaluation of a state is provided by the best possible decision d^* to be taken, which first maximizes the resulting probability of feasibility $f_1(t, V_B, V_S)$ and then minimizes the expected total turnaround time $f_2(t, V_B, V_S)$. At any given decision time, for a non-final state, the decisions are numbered $0, 1, \ldots, d, \ldots, |V_B|$. The decision 0 is to delay the service of all waiting ships, in order to wait for the arrival of another ship. The decision $d \geq 1$ is to start the service of the ship number d.

At any decision time t, if all ships have arrived ($V_S = \emptyset$), the evaluation of the corresponding state is immediate as the shortest processing time rule is optimal. Otherwise, for a non-final state, the decision 0 of waiting is evaluated as follows:

$$f_1^0(t, V_B, V_S) = \sum_{S \subseteq V_S} \Pr(S, t+1) f_1(t+1, V_B \cup S, V_S \setminus S), \tag{3}$$

$$f_2^0(t, V_B, V_S) = \sum_{S \subseteq V_S} \Pr(S, t+1) f_2(t+1, V_B \cup S, V_S \setminus S). \tag{4}$$

For a waiting decision, the probability $\Pr(S, t+1)$ of the arrival of the subset of ships $S \subseteq V_S$ at time $t+1$ is:

$$\Pr(S, t+1) = \prod_{v \in S} \frac{\Pr(r_v = t+1)}{\Pr(r_v \geq t+1)} \prod_{v \in (V_S \setminus S)} \left(1 - \frac{\Pr(r_v = t+1)}{\Pr(r_v \geq t+1)}\right). \tag{5}$$

Finally, the decision of serving ship $d \in [\![1, |V_B|]\!]$ is evaluated as follows:

$$f_1^d(t, V_B, V_S) = \sum_{S \subseteq V_S} \Pr(S, t+1, t+p_d) f_1(t+p_d, V_B \setminus \{d\} \cup S, V_S \setminus S), \tag{6}$$

$$f_2^d(t, V_B, V_S) = \sum_{S \subseteq V_S} \Pr(S, t+1, t+p_d) f_2(t+p_d, V_B \setminus \{d\} \cup S, V_S \setminus S) + t + p_d. \tag{7}$$

For the decision of serving ship d, the probability $\Pr(S, t+1, t+p_d)$ of the arrival of the subset of ships $S \subseteq V_S$ between times $t+1$ and $t+p_d$ is:

$$\Pr(S, t+1, t+p_d) = \prod_{v \in S} \frac{\Pr(t+1 \leq r_v \leq t+p_d)}{\Pr(r_v \geq t+1)} \prod_{v \in (V_S \setminus S)} \left(1 - \frac{\Pr(t+1 \leq r_v \leq t+p_d)}{\Pr(r_v \geq t+1)}\right). \tag{8}$$

The states are evaluated starting from time $\max\{r_v^{\max} \mid v \in V\}$, down to time $\max\{s_b, \min\{r_v^{\min} \mid v \in V\}\}$.

4 Numerical experiments

We conduct numerical experiments, with two main objectives. The first objective is to evaluate the efficiency of the proposed approach to the stochastic berth allocation problem. The second objective is to estimate the value of stochastic information for this problem. The usual deterministic approach is taken as reference, consisting in solving exactly the deterministic planning problem with average arrival times.

The instances of the benchmark set are adapted to the case of stochastic arrival times. There are six types of instances: 25x5 (25 ships and 5 berths), 25x7, 25x10, 35x7, 35x10 and 60x12. There are ten instances of each of the five first types and thirty instances of the last type. The arrivals are distributed over a period of about 6 days. The probability distribution of each arrival time is set to a binomial distribution, with 73 trials and a probability of success equal to 0.5. Such a binomial distribution approximates a normal distribution, with a standard deviation equal to 4.2 hours.

Instance	Stochastic planning		Deterministic planning	
	time	opt. gap	time	opt. gap
25x5	115.0	3.9	6.0	4.4
25x7	94.9	3.3	6.7	3.8
25x10	69.1	2.1	6.6	3.0
35x7	199.4	4.5	22.4	4.9
35x10	152.9	3.3	19.5	3.7
60x12	286.9	2.4	9.5	3.6
Average	186.5	3.0	11.2	3.8

TAB. 2: Comparative numerical results: stochastic planning approach against the deterministic one

The approaches to the stochastic berth allocation problem are evaluated with 1000 generated scenarios for each instance. To assess the approximations provided by the restricted sets of scenarios, we compared the exact and scenario-based evaluations of the computed plannings. They always differ by less than 0.15%, which show that the approximations are reliable.

The deterministic planning problem associated with each scenario is solved optimally, providing the optimal value of the scenario. The optimal value of an instance in the stochastic case is approximated by the average optimal value of its 1000 scenarios. It corresponds to the case where optimal decisions of berth assignment and ship sequencing are taken in each scenario.

In table 2, we report comparative numerical results between the stochastic planning approach (column Stochastic planning) and the deterministic one (column Deterministic planning). As each of the approaches provided a feasible solution to each of the generated scenarios, only the minimization of turnaround time is considered. Each line of the table reports aggregated results by type of instances. The average running time in seconds (column time) and the average optimality gap in percent (column opt. gap) are provided.

In the case of stochastic arrival times, the deterministic planning approach performs relatively well, with a global average optimality gap of 3.8%. Still, the stochastic planning approach reduces this gap to 3%, which is indeed a relative improvement of 21%. The numerical results of the optimal dynamic management of each berth will be presented, obtained with the stochastic dynamic programming algorithm. With the instance types with the highest ships to berths ratios (25x5, 35x7, 60x12), an additional relative improvement of about 5% is observed. Moreover, this algorithm significantly outperforms the fixed sequences of the planning in some worst-case scenarios with unfavorable orders of arrival.

References

[1] Christian Bierwirth and Frank Meisel. “A follow-up survey of berth allocation and quay crane scheduling problems in container terminals”. In: *European Journal of Operational Research* 244.3 (2015), pp. 675 –689.

[2] Jean-Francois Cordeau, Gilbert Laporte, Pasquale Legato, and Luigi Moccia. “Models and Tabu Search Heuristics for the Berth-Allocation Problem”. In: *Transportation Science* 39.4 (2005), pp. 526–538.

[3] Nitish Umang, Michel Bierlaire, and Alan L. Erera. “Real-time management of berth allocation with stochastic arrival and handling times”. In: *Journal of Scheduling* (2016), pp. 1–17.

[4] Bert Vernimmen, Wout Dullaert, and Steve Engelen. “Schedule unreliability in liner shipping: origins and consequences for the hinterland supply chain”. In: *Maritime Economics & Logistics* 9.3 (2007), pp. 193–213.