Domain Clustering for Inter-Domain Path Computation Speed-Up

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1 Scenario

Due to scalability and privacy issues, large networks are often partitioned into several domains, e.g., Interior Gateway Protocol areas. In order to limit the number of Label Switched Paths crossing domains, a crucial constraint for inter-domain path computation (IDPC) problem is *Domain Uniqueness* (DU), i.e., paths can cross a domain at most once [1],[2]. Typically, the path computation inside each domain is managed by a Path Computation Element (PCE). Mainly two architectures, distributed and hierarchical, have been envisioned for handling communication among PCEs. In general, distributed architectures fail to solve the multi-domain path computation problem optimally, hence we consider the hierarchical PCE (h-PCE) architecture [1]. A Parent PCE sits on top of the hierarchy, gathering *intra*-domain routing information from all Children PCEs to perform *inter*-domain path computation. On the other hand, h-PCE is affected by serious scalability issues when the number of domains increases [2], due to two main bottlenecks, namely 1) Parent PCE's computational capabilities and 2) the capacity of the Children-to-Parent PCE communication channel. Therefore, we propose a new concept allowing to artificially cluster domains during a pre-processing phase.

2 Inter-Domain Path Computation and Domain Clustering

We model a generic multi-domain network as a weighted colored directed multi-graph $\mathcal{G} = (V, E)$, where V is the set of nodes and E is the set of edges. Let $(i, j)^k \in E$ denote the k-th parallel edge between nodes i and j. Each edge $(i, j)^k$ has an associated positive cost $w^k_{i,j}$ and a color $d \in \mathcal{D}$, where $\mathcal{D}$ is the finite set of colors. Let E^d be the set of edges having color d. We interpret the color of an edge as the network partition, called *domain*, which the edge belongs to. We say that a path $\mathbf{p}$ is *feasible* if it does not revisit a domain that it has already left. More formally, for all i, d such that if $p_i \in E^d$ and $p_{i+1} \notin E^d$, then $p_{i+k} \notin E^d$, for all $k \geq 2$. We denote $\mathcal{F}_{s,t}(\mathcal{G})$ as the set of feasible paths on graph $\mathcal{G}$ from source node s to destination t. We then wish to solve the following IDPC-DU optimization problem :

$$(\mathrm{IDPC-DU}) \qquad \mathbf{p}^* = \underset{\mathbf{p}\in\mathcal{F}_{s,t}(\mathcal{G})}{\mathrm{argmin}} \sum_{(i,j)^k\in\mathbf{p}} w^k_{i,j} = w(\mathbf{p}).$$

We first prove the hardness of IDPC-DU.

Theorem 1. *The decision problem associated with IDPC-DU is NP-complete.*

As a corollary, we can claim that, unless P = NP, IDPC-DU is *not approximable* in polynomial time. Although IDPC-DU is hard, we wish to find a pre-processing technique that allows to reduce its complexity at run-time. To this aim, we first propose a dynamic programming

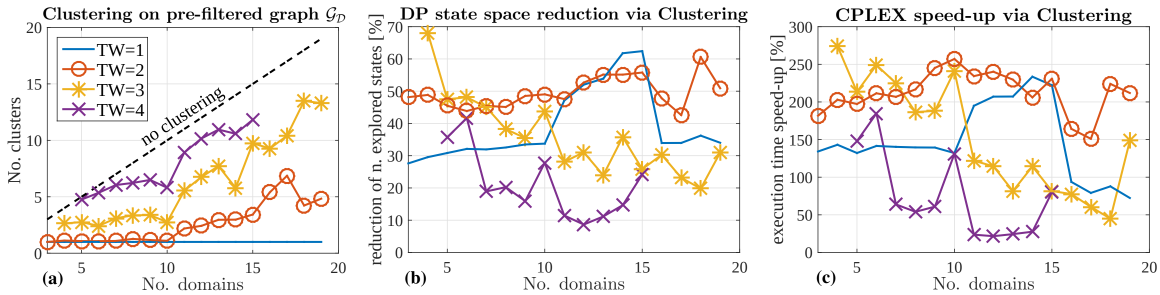

FIG. 1 – First, $\mathcal{G}_\mathcal{D}$ (undirected and strongly connected) is pre-filtered by removing edges not on acyclic paths. Then, a proper clustering function i found. **(a)** No. of clusters vs. No. of domains, for different values of treewidth of the inter-domain graph $\mathcal{G}_\mathcal{D}$. **(b-c)** Complexity reduction/speed-up brought by clustering to DP (b) and CPLEX (c) solutions, on top of pre-filtering.

(DP) algorithm to solve IDPC-DU whose states comprise information on the current node of the path, the traversed domains and the current domain. Its complexity is $O(|V|^2 2^{|\mathcal{D}|}|\mathcal{D}|^2)$. This suggests that we should look for a way to artificially decrease the number of domains to also decrease the complexity of IDPC-DU. Let h be a function mapping domains to clusters. We say then domains in $h^{-1}(c) \subseteq \mathcal{D}$ are *aggregated* in the cluster c. Next, let $\mathcal{G}^h$ be a colored weighted graph having same nodes, edges and costs as $\mathcal{G}$, except for the color of edges which are clustered according to clustering function h. We say that h is *proper* whenever IDPC-DU can be equivalently solved on the original graph $\mathcal{G}$ or on the (reduced) clustered graph $\mathcal{G}^h$, i.e.,

$$\arg\min_{\mathbf{p}\in\mathcal{F}_{s,t}(\mathcal{G})} w(\mathbf{p}) = \arg\min_{\mathbf{p}\in\mathcal{F}_{s,t}(\mathcal{G}^h)} w(\mathbf{p}), \qquad \forall\, s,t \in V.$$

Finally, we define the inter-domain graph $\mathcal{G}_\mathcal{D} = (\mathcal{D}, E_\mathcal{D})$, which has a direct link $(d,q) \in E_\mathcal{D}$ iff there exists a path on the original graph $\mathcal{G}$ going from domain d to q. We note that $\mathcal{G}_\mathcal{D}$ is a succinct description of the network, being also stable to link/node failures. We are now ready to provide a necessary and sufficient condition under which h is a proper clustering function, which then allows to reduce the complexity of IDPC-DU by artificially reducing the number of domains without loss of optimality.

Condition 1 (Intra/Extra-Cluster Condition (IECC))**.** *Let h be a clustering function and let $c \in h(\mathcal{D})$ be a cluster. Let $\mathcal{G}_\mathcal{D}(h^{-1}(c))$ be the subgraph of $\mathcal{G}_\mathcal{D}$ restricted to aggregated domains $h^{-1}(c) \subseteq \mathcal{D}$. Then, for every cluster $c \in h(\mathcal{D})$ the following two conditions are jointly satisfied :*

i) (Intra-Cluster) *The subgraph $\mathcal{G}_\mathcal{D}(h^{-1}(c))$ is acyclic ;*

ii) (Extra-Cluster) *After removing all edges in $\mathcal{G}_\mathcal{D}(h^{-1}(c))$ from $\mathcal{G}_\mathcal{D}$, all pairs of different domains belonging to $h^{-1}(c)$ are disconnected.*

Theorem 2 (IECC sufficiency)**.** *If a clustering function h fulfills IECC, then h is proper.*

Theorem 3 (IECC necessity)**.** *Let $\{d_1, d_2, \ldots, d_k\}$ be a walk on $\mathcal{G}_\mathcal{D}$, where $d_i \neq d_j$ for all $i \neq j$, except possibly for $i = 1$ and $j = k$. Suppose that there exists a simple path on $\mathcal{G}$ that visits domains $\{d_1, d_2, \ldots, d_k\}$ in this order. Then, IECC is also necessary for h to be proper.*

In Fig. 1 we show the complexity reduction achieved via domain clustering for IDPC-DU.

Références

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