The Inventory-Routing Problem with Transshipment

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1 Introduction

Since its successful application in the late 1980's by Wal-Mart and Procter & Gamble, Vendor-Managed Inventory (VMI) has gained enormously in popularity [1]. Currently, it is one of the most discussed supply chain strategies and research on the topic shows its potential for improving supply chain performance [2]. Suppliers that are engaged in a VMI system should integrate inventory management and transportation planning and take decisions to optimize the costs of both subsystems simultaneously. The mathematical problem that supports these decisions is known as the Inventory Routing Problem (IRP).

In this research, we deal with a specific variant of the IRP : the Inventory-Routing Problem with Transshipment (IRPT). In this problem not only the supplier can deliver goods to the customers, but also transshipments between customers or between the supplier and a customer are possible. These transshipments are carried out by a subcontractor. The objective is to determine the quantities delivered to all customers by both the supplier and the subcontractor in the different periods of the planning horizon while minimizing the total cost.

It is worthwhile to study the IRPT, from both an industrial and academic point of view. In a recent and comprehensive survey on the industrial aspects of inventory-routing, Andersson et al. [4] indicate the need for richer and more flexible models. The IRPT offers a great opportunity to build more flexible models. By including the subcontractor it also shows the increased collaboration that exists between the actors of the supply chain in reality.

2 Model

2.1 Valid Inequalities and bounds

After the introduction of the IRPT a number of valid inequalities have been proposed in Coelho and Laporte [3]. Starting from these inequalities we derive new inequalities for the problem. The derived inequalities act mostly on the inventory management part of the problem. We demonstrate that adding other inequalities that act on the routing part or on the combined inventory-routing part of the problem often result in a trade-off between the strength of the linear relaxation and the speed of the branch-and-cut procedure.

We also strengthen the lower and upper bounds of the inventory variables and the transportation variables. We demonstrate how this results in the simplification and the elimination of a number of constraints.

2.2 Dantzig-Wolfe Decomposition

Next to the addition of new valid inequalities, we were also able to reformulate the problem using the Dantzig-Wolfe decomposition framework. The resulting pricing subproblem is an NP-complete problem for which currently no efficient solution method is known. Therefore, the Dantzig-Wolfe decomposition is not suited to solve the IRPT to optimality. However, the Dantzig-Wolfe linear relaxation provides a good lower bound on the problem. We also demonstrate how this bound can be improved by adding the new valid inequalities to the Dantzig-Wolfe decomposition and how the solution time of the pricing subproblem can be accelerated by adding certain particular solutions a priori to the column generation algorithm.

2.3 Preliminary Results

Our preliminary results show that adding the valid inequalities accelerates the solution time of the benchmark instances in literature. It also shows that the Dantzig-Wolfe decomposition linear relaxation provides a good lower bound on the problem.

3 Conclusions and perspectives

The preliminary results confirm the interest of adding valid inequalities to the formulation of the IRPT. We believe that further investigation of the routing part and the combined inventory-routing part of the problem will lead to stronger formulations which accelerate the branch-and-but procedure even more.

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