A Column Generation approach for a Multi-Activity Tour Scheduling Problem

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1 Introduction

Personnel scheduling problems consists of constructing feasible shift schedules to be assigned to company staff, in order to satisfy workload requirements. These problems arise in several industries and organizations of different kinds. Due to economic considerations, personnel scheduling represents an intense and challenging research field [1]. Three main categories of problems can be distinguished in personnel scheduling : shift scheduling, days-off scheduling and tour scheduling. In this work we deal with a problem in the latter category. We aim to specify the time periods of the day and the days of the week in which employees have to work. Moreover, when more than one work activity has to be scheduled, the problem becomes a Multi-Activity Tour Scheduling problem. In these problems, we need not only to define the working days and the working periods, but also to specify the allocation of work activities. Several methods can be found in the literature [1] to solve this problem. We propose a column generation approach, where the reduced master problem is not solved exactly by means of an LP solver, but through a dual ascent heuristic that estimates dual optimal variables. Our approach is able to soften some effects of which column generation suffers, such as the oscillation of dual variables and the tailing-off effect.

2 Column generation approach

Column generation is a classical technique to solve a linear program by iteratively adding the variables of the model, which makes this technique interesting for problems with a large number of variables. Recently, column generation approach has been widely used to solve multi-activity tour scheduling problems [2], [3]. This method is based on the decomposition of the problem into restricted master problem and subproblems, that are solved iteratively. We consider on one hand a master problem that takes into account workload requirements, minimizing the total cost given by under and over coverage. On the other hand, new feasible schedules (columns) are built solving the subproblems where we consider the legal constraints, such as consecutive working hours, daily working hours, break duration, etc.

The master problem has the following formulation :

$$\min \sum_{i\in\mathcal{I}} \sum_{p\in P_i} c_p\lambda_p + \sum_{j\in J}\sum_{k\in K} \left(\underline{c}_{jk}\, u_{jk} + \bar{c}_{jk}\, o_{jk}\right) \tag{1}$$

$$\sum_{i\in\mathcal{I}} \sum_{p\in P_i} x_{pjk}\,\lambda_p + u_{jk} - o_{jk} = d_{jk}, \qquad \forall j \in J, \forall k \in K, \tag{2}$$

$$\sum_{p\in P_i} \lambda_p = 1, \qquad \forall i \in I, \tag{3}$$

$$u_{jk}, o_{jk}, \lambda_p \geq 0, \qquad \forall p \in P_i, \forall i \in I, \forall j \in J, \forall k \in K, \tag{4}$$

where I is the set of employees, P_i is the set of columns assignable to employee i, J is the set of time slots, and K is the set of activities. The coefficients c_p, $\underline{c}_{jk}$ and $\bar{c}_{jk}$ are respectively the costs of columns p and the costs of under and over coverage of activity k in slot j. Coefficient x_{pjk} is equal to 1 if the activity k is assigned in slot j in column p, otherwise it is equal to 0. Variables u_{jk} and o_{jk} represent respectively the under and over coverage for activity k in slot j. Problem (1)-(4) is the linear relaxation of the integer linear problem where variables λ_p are binary. The goal of such integer problem is to assign one schedules to each employee (3), satisfying the workload requirements (2), and minimizing the total cost given by the sum of the schedules' costs and the under-over coverage costs (1).

3 Dual ascent heuristic

The master problem appears in a set-partitioning like formulation. We do not solve exactly the problem by means of an LP solver. We adapted the dual ascent heuristic proposed by [4] for set partition problems, in order to take into account right hand side vectors different from the unit vector, under coverage and over coverage. This procedure is based on a parametric relaxation, and makes use of Lagrangian relaxation and subgradient method. The lower bound achieved is better than the one found by the classical Lagrangian relaxation. The dual variables produced by the heuristic are then used to define the objective function of the subproblems. Computational results show that our approach is able to soften some effects of which column generation suffers, such as the oscillations of dual variables and the tailing-off effect.

4 Conclusions et perspectives

We proposed a column generation approach to solve the multi-activity tour scheduling problem. Our method makes use of a dual ascent heuristic to solve the reduced master problem and to obtain a good estimation of dual variables. Preliminary results show an interesting decrease of tailing-off and oscillation effects. We aim to develop a complete method able to find an integer solution of the problem (1)-(4) considered.

Références

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