On the b-domatic partition of some graphs

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1 Introduction

Let $G = (V, E)$ be a finite, simple and undirected graph with vertex-set V and edge-set E. We call $|V|$ the order of G and denote it by n. For any nonempty subset $A \subset V$, let $G[A]$ denote the subgraph of G induced by A. For any vertex v of G, the *neighborhood* of v is the set $N_G(v) = \{u \in V(G) \mid (u, v) \in E\}$ and the *closed neighborhood* of v is the set $N_G[v] = N_G(v) \cup \{v\}$. Let $\Delta(G)$ (respectively, $\delta(G)$ be the maximum (respectively, minimum) degree in G. Through this paper, the notations P_n, C_n, and K_n always denote a path, a cycle, and a complete graph of order n, respectively, while $K_{p,q}$ $(p \geq q)$ denotes a complete bipartite graph with partite sets of sizes p, q . For further terminology on graphs we refer to the book by Berge [2].

A set $S \subseteq V$ is called a dominating set if every vertex in $V \setminus S$ is adjacent to some vertex in S. The minimum cardinality of a dominating set is called the domination number and is denoted by $\gamma(G)$. By analogy to the concept of chromatic partition, Cockayne and Hedetniemi [3] introduced the concept of domatic partition of a graph. A partition $\mathcal{P}$ in which each of its classes is a dominating sets is called a domatic partition of G. The domatic number $d(G)$ is defined as the largest number of sets in a domatic partition. The authors of [3] showed that

$$d(G) \leq \min\{\frac{n}{\gamma(G)}, \delta(G) + 1\}. \quad (1)$$

For some other results on domatic partition see [1, 4, 6].

In [5], O. Favaron introduced the b-domatic number by considering a new type of domatic partition. As defined in [5], a domatic partition $\mathcal{P}$ of G is *b-domatic* if no larger domatic partition π can be obtained by transferring some vertices of some classes of $\mathcal{P}$ to form a new class. Formally, a partition $\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_k\}$ is a b-domatic partition of G if there do not exist k non-empty subsets $\pi_i \subseteq \mathcal{C}_i$, $i \in \{1, .., k\}$ with strict inclusion for at least one non-empty subset $(\mathcal{C}_i \setminus \pi_i \neq \emptyset)$ for which $\pi = \{\pi_1, \pi_2, ..., \pi_k, V \setminus \bigcup_{i=1}^{k} \pi_i\}$ is domatic partition of G. The minimum cardinality of a b-domatic partition of G is called the b-domatic number and is denoted by $bd(G)$.

It is observed in [5] that if $\delta(G) = 0$, then $\{V(G)\}$ is the unique domatic partition and so $bd(G) = d(G) = 1$. For this, all graphs considered in this paper are without isolated vertices. Many other properties of domatic and b-domatic partitions were given in [5]. In particular, it was observed that for any graph G with degree minimum $\delta(G) \geq 2$,

$$2 \leq bd(G) \leq d(G). \tag{2}$$

In this paper, we investigate new properties of a b-domatic partition. Firstly, we give necessary and sufficient conditions for which a given domatic partition of a graph G is b-domatic. Next, we present some classes of graphs for which $bd(G) = 2$ and $bd(G) = \delta(G) + 1$. Other results are given for particular classes of graphs.

2 Mains results

Theorem 1 *Let $\mathcal{P}$ be a domatic partition of a graph $G = (V, E)$. Then $\mathcal{P}$ is b-domatic if and only if there exists a vertex $x \in V$ such that each vertex of $N_G[x]$ is either isolated in its class or has a private neighbor with respect to its class.*

Corollary 2 *Let $\mathcal{P}$ be a domatic partition of a graph G. If every vertex x of G is either isolated in its class or has a private neighbor with respect to its class of x, then $\mathcal{P}$ is b-domatic.*

Proposition 1 *Let H be a graph. If G is the prism of H or the complementary prism of H, then $bd(G) = 2$.*

Corollary 3 *The Petersen graph P satisfies $bd(P) = 2$.*

Proposition 2 *Let G be a block graph of minimum degree $\delta(G)$. Then $bd(G) = \delta(G) + 1$.*

Theorem 4 *Let $G = (V, E)$ be a r-regular graph and $\mu = \max\{|S_x| : x \in V(G)$, S_x is a maximum independent set of $G[N(x)]\}$. If $d(G) = r + 1$, then $bd(G) \leq r - \mu + 2$.*

A vertex x of a graph G is universal if it is adjacent to every other vertex of G. It was showed in [4] that if x is a universel vertex of a graph G, then $d(G) = d(G - x) + 1$. We give her a similar result for the b-domatic number.

Proposition 3 *If x is universal vertex in G, then $bd(G) = bd(G \setminus x) + 1$.*

Références

[1] S. Arumugam, K. Raja Chandrasekar, Minimal dominating sets in maximum domatic partitions, Australasian Journal of Combinatorics, V. 52 (2012), Pages 281–292.

[2] C. Berge. *Graphs*. North Holland, 1985.

[3] E.J. Cockayne and S.T. Hedetniemi, Towards a theory of domination in graphs, Networks 7 (1977) 247–261.

[4] G. J. Chang. The domatic number problem, Discrete Mathematics 125 (1994) 115-122.

[5] O. Favaron. The b-domatic number of a graph, Discussiones Mathematicae, Graph Theory 33(2013)747–757.

[6] S-H. Poon, W. C-K. Yen, C-T. Ung, Domatic Partition on Several Classes of Graphs, Combinatorial Optimization and Applications, V. 7402 of the series Lecture Notes in Computer Science pp 245-256.