Service Level in Scheduling: Case of the Two-Machine Permutation Flow-Shop Problem

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Mots-clés : *Operations Research, Stochastic Scheduling, Two-Machine Permutation Flow-Shop, Uncertainty, Allen's Algebra*

1 Introduction

Stochastic scheduling problems, in which job processing times are supposed to be stochastic, have been mainly studied since the 1980's in accordance with different formats of uncertain data representation: interval-based, possibilistic and probabilistic. These problems are able to better meet some industrial realities than their deterministic counterparts, since they allow the inherent uncertainties present in many real-world manufacturing environments to be captured and taken into account [4].

As far as probabilistic-based scheduling problems are concerned, the optimization of the mathematical expectation of a given criterion appears to be the most common approach in the related literature. As highlighted in [3], optimizing the mean of a given criteria is rather insensitive to the intrinsic variability of uncertain data.

Another (more subtle) manner to deal with probabilistic information, which helps to balance between *conservatism* and *performance*, is to operate with a given service (confidence) level. Different definitions for the service level can be encountered in the literature. In particular, [3] introduced the concept of *service level* in scheduling, as the probability that a criterion is smaller (or larger) than or equal to an imposed value $(1-\alpha)$. Moreover, the authors of [3] pointed out the relevance of this concept from a practical point of view.

Following the work of [3], this paper focuses on the stochastic two-machine permutation flow-shop scheduling problem, by handling explicitly random job processing times, which are defined on a finite support with known probability distributions. More specifically, we investigate the relationship between the optimal values of the makespan and the associated service levels $(1-\alpha)$, which allows us to better model the impact of the variability of uncertain data.

2 Analytical study

Let us consider the probabilistic non-preemptive two-machine flow-shop problem in which the makespan is minimized. There are n jobs j to be processed by two machines $i=\{1,2\}$. Assume that p_{ij} is the random variable corresponding to the processing time of job j on machine i, defined by a cumulative probability distribution F_{ij} on a finite support $[p_{ij}^L, p_{ij}^U], \forall j=\overline{1,n}, \forall i=\{1,2\}$. Restricting the domain of probability distributions to a finite range is relevant for both theoretical and practical reasons, since considering processing times beyond $\pm 3\sigma$ (or other truncation points) is most often not necessary.

Prior to this work, [5, 6] addressed the $F_2|prmu, p_{ij}^L \leq p_{ij} \leq p_{ij}^U|C_{\max}$ problem, by looking for a minimal set of the dominant schedules restricted by the Johnson's permutations. Always referring to the Johnson's rule, various sufficient conditions have also been identified for the precedence of jobs. More general sufficient conditions for the optimality of a permutation of jobs are proven in [2], where a sufficient condition of optimality for the $F_2|prmu|C_{\max}$ is established drawn on two interval structures associated to the problem. This optimality sufficient condition describes a set of optimal solutions, by determining the bases and the corresponding b-pyramids.

Using the Allen's algebra [1] and based on the sufficient condition provided by [2], the contribution of this paper is multiple:

- ❒ Generalize the properties obtained by [5, 6] for the case when probability distributions describing random job processing times are known;
- ❒ Extend the results on the characterization of optimal solutions for $F_2|prmu|p_{ij}^L \leq p_{ij} \leq p_{ij}^U|C_{\max}$ provided by [5];
- ❒ Consider that probability distributions between the bounds of processing times are available and propose valuable properties for the problem $F_2|prmu, p_{ij} \sim F_{ij}|C_{\max}$ while calculating the corresponding service level $(1-\alpha)$.

3 Perspectives

Through the prism of the stochastic flow-shop problem $F_m|prmu, p_{ij}^L \leq p_{ij} \leq p_{ij}^U|C_{\max}$, our future work will be addressing the two open questions in [3]:

- ❒ "How to determine the maximum service level corresponding to a given schedule, which respects a bounded value of the objective function?";
- ❒ And "How to determine the schedule that optimizes the service level, while respecting a given value of the objective function?".

In order to achieve this goal and given the complexity of the computation of multivariate integrals, we intend to solve these problems by discretizing/randomizing probability distributions and exploiting the concept of p-efficiency.

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