Exact and approximation algorithms for multi-level in series lot-sizing problems with batch deliveries

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Presentation of the problem

We consider the distribution of a single item through a logistics network constituted of N levels in series. The network is facing a deterministic demand at the last level, over a time horizon divided into T periods. At each period t, demand d_t must be satisfied on time, that is, neither lost sale nor backlogging are allowed. Level N can satisfy the demands from its stock or/and by ordering some units from level $(N-1)$, which in its turn can order from its upstream level $(N-2)$, and so on. We consider that level 1 orders the units from an external supplier. The objective is to serve all the demands at the minimum cost. There are two kind of costs in the system: An unit holding cost h_t^i is paid to carry one item in stock from period t to $t+1$ at level i, while a procurement cost is paid to order units from upstream. We consider in this paper that orders are delivered by batch of size B: in addition to a unit ordering cost p_t^i, a fixed cost k_t^i is charged for each batch. The cost of ordering x units at level i in period t is thus equal to $p_t^i x + \lceil x/C \rceil k_t^i$. The multi-level uncapacitated lot-sizing problem with batch deliveries can model a supply chain where items are transported using identical shipping containers or trucks. As far as we know, the status of this problem is open when the number N of levels is part of the inputs.
We propose the following formulation, where decision variables x_t^i represent the amount ordered at period t at level i and s_t^i is the stock level at the end of period t at level i.

$$\min \sum_{t=1}^{T} \sum_{i=1}^{N} (\lceil x_t^i/B \rceil k_t^i + p_t^i x_t^i + s_t^i h_t^i) \tag{P}$$

subject to

$$x_t^i + s_{t-1}^i = x_t^{i+1} + s_t^i, \ \ \forall t \in \{1,..,T\}, \, i \in \{1,..,N-1\}, \tag{1}$$

$$x_t^N + s_{t-1}^N = d_t + s_t^N, \ \ \forall t \in \{1,..,T\}, \tag{2}$$

$$s_0^i = 0, \ \ \forall i \in \{1,..,N\}, \tag{3}$$

$$x_t^i \geq 0, \ \ \forall t \in \{1,..,T\}, \forall i \in \{1,..,N\} \tag{4}$$

$$s_t^i \geq 0, \ \ \forall t \in \{1,..,T\}, \forall i \in \{1,..,N\} \tag{5}$$

We establish that this multi-level lot-sizing problem can be solved in polynomial time assuming that: (i) unit ordering and holding costs follow a *non-speculative motives echelon cost structure*. At

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the first level the assumption is similar to that used in single-level problems: $p_t^1 + h_t^1 \geq p_{t+1}^1$ must hold for any period $t \in \{1, ..., T-1\}$. At the other levels, the formula is slightly modified to take into account the holding cost saved at the previous level: $p_t^i + (h_t^i - h_t^{i-1}) \geq p_{t+1}^i$, for all $i \in \{2, ..., N\}$, $t \in \{1, ..., T-1\}$ (ii) at each level, the fixed costs per batch are non-increasing with time

Decomposition of a solution

Our algorithm is based on a set of dominant properties, which allows us to exhibit a particular structure of extreme points. More precisely, we show that each full batch ordered at a period t and a level i is also ordered in t at the higher levels $(1, ..., i-1)$. Similarly, if a fractional batch is ordered at a period t and a level i, every lower levels will also order a fractional batch.
We then define the key structure of our algorithm, called a Box, using the flow representation of an optimal solution observing all dominant properties that we have highlighted. A Box denoted $B^i(r, s)$ is a connected component induced by levels $(i, i+1, .., N)$ and such that all its nodes are located between periods r and s. We show that two consecutive boxes on the same levels do not overlap. We are able to evaluate the amount of the flows in a Box $B^i(r, s)$ independently for given values of i, r and s, by decomposing $B^i(r, s)$ into a set of boxes on levels $(i+1, ..., N)$. Using the concept of basis path introduced by Hwang et al. (2013)[1], the flow over level i can be determined as well as the amount of the fractional orders at level $i+1$ between periods r and s. In other words, a Box $B^i(r, s)$ can be decomposed into a set of boxes having one less level than $B^i(r, s)$, and the amount stored and ordered at level i results from this decomposition. The same process is thus performed for each Box on levels $(i+1, ..., N)$ which are in turn decomposed into boxes on level $(i+2, ..., N)$. The process is repeated until the last level is reached.

Theorem 1 *The multi-level lot-sizing problem with our cost structure can be solved in time complexity $O(NT^3)$.*

Extension: time-dependent/level dependent batch sizes

We extend the problem to the case with batch deliveries when batch sizes are time-dependent at each level (B_t^i). In addition, we consider a non-null setup cost K_t^i for ordering at a period t at a level i. Notice that this problem is NP-hard even in its single-level version[2].

Theorem 2 *The multi-level lot-sizing problem with batch deliveries is NP-hard when batch sizes are non-stationary.*

We propose a simple approximation algorithm. The cost of ordering x units is $q_t^i(x) = p_t^i x + K_t^i + \lceil x_t^i / B \rceil k_t^i$. The approximation algorithm consists of solving the problem without considering the batches $(B_t^i = +\infty)$, and using the following affine ordering cost: $r_t^i(x) = (K_t^i + k_t^i)/2 + (p_t^i + k_t^i/2)x$. It can be done in $O(NT^4)$ with the algorithm of Zangwill (1969)[3]. The resulting solution is feasible for the original problem and is lower than twice the value of the optimum since $r_t^i(x) \leq q_t^i(x) \leq 2r_t^i(x)$. We obtain a $2-$approximation algorithm in $O(NT^4)$ for the multi-level lot-sizing with batch deliveries.

[1] Hwang, H.-C. and Ahn, H. and Kaminsky, P., Basis Paths and a Polynomial Algorithm for the Multistage Production-Capacitated Lot-Sizing Problem, Operations Research, 61, 469–482 (2013)

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