New MIP models for Liner Shipping Route Design with Empty Container Repositioning

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1 Main features of the route design problem

This paper addresses the problem of optimal planning of a line for a container shipping company. Given estimated weekly splittable demands between pairs of ports and bounds for the turnaround time, our goal is to determine the subset of ports to be called and the amount of containers to be shipped between each pair of ports, so as to maximize the profit of the shipping company, that is, revenue minus operational costs. In order to save possible leasing or storage costs of empty containers at the respective ports, our approach takes into account the repositioning of empty containers. We assume a predetermined ordering of ports for the outbound-inbound trips, which is a natural way of scheduling routes in the inland waterway transport, along a coastline, or on routes with given directions (e.g., East-West trades). The port calling sequence must start at and return to the first port (in case of barge transport, it is a sea port, located at a river mouth). The liner ship must stop at the last port before going back to the first port. Moreover, the ship must complete its route within a given time interval.

Liner shipping network design under these assumptions has been introduced in [3]. Since then, these concepts have been extended by introducing new and relevant aspects for maritime or inland waterway shipping. A recent survey on maritime route design problems can be found in [2]. The challenging question studied in this paper is: how to develop an integrated approach to design shipping routes while taking into account empty container balancing and repositioning at the same time?

To our knowledge, the first integrated approach involving empty container repositioning was considered in [4]. A problem variant for barge container shipping with outbound-inbound principle and empty container repositioning has been studied in [1]. The present research covers some of the main features of liner container route design with empty container balancing and repositioning that have been partially studied in the above papers.

2. **Contribution**

In this article, we propose two new MIP formulations that explicitly take advantage of the outbound-inbound principle. In contrast to the existing models from the literature (that require arc-variables for modeling the routes), both new models use node-variables for the route design. The first formulation requires arc-variables for modeling empty containers. The second formulation is more compact as it uses node-variables only and handles empty containers as a single commodity. An equivalence of the two models, concerning the strength of their LP relaxations, is shown. The two models first apply to the case where the distance traversed between the starting and last port remains constant whatever the port calling sequence, as this is the case for barge container shipping along a river. We then extend our models to the general case of [3] where shortcuts between ports are allowed**.** We furthermore show that the problem remains strongly NP-hard, even after relaxing many of its constraints, and we also discuss a special polynomially solvable case.

Finally, we show how to extend the new MIP models so as to (1) optimize the size of the fleet and maximize the profit simultaneously, (2) find the optimal number of round-trips within the planning horizon, or (3) deal with unsplittable demands. With the extension (2) we are addressing the same problem as the one described in [3], while additionally taking the empty container balancing and repositioning into account.

Our computational study is conducted on a set of benchmark instances of barge container shipping from the literature. Our new modeling based on node-variables for route design enables us to significantly reduce computation time and to solve to optimality all instances with up to 25 ports in a few seconds only, thus drastically outperforming the previous state-of-the-art model. For the more challenging variant with unsplittable demands, our approach is able to compute near-optimal solutions within a short computing time.

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