

# Designing Stochastic Two-Echelon Distribution Networks

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## 1 Problem context

Delivering manufacturing goods from production platforms to demand zones is often managed through one or more intermediate locations where storage, transshipment and consolidation activities are performed. When distribution activities cover a large geographical area, depending on hierarchical inventory rules, or concerning urban deployment, multi-echelon network configurations are more appropriate [1]. This type of configuration allows considerable savings compared to direct deliveries from one main depot : intermediate location help to reduce the many freight vehicles going into cities.

In this work, a more realistic and complex distribution design problem is addressed which has received limited attention in the literature. It involves facility location and transportation decisions. More precisely, the transportation decisions consist in the transportation from platforms to intermediate locations, and from intermediate locations to demand zones. The origin-destination flows trigger design variables to define the number and the location of the intermediate facilities that form the two-echelon distribution network. A multi-year planning horizon is considered, where demand varies dynamically from one year to the next. Location capacities can be selected tuned to the needs that evolve during the horizon. Thus, the two-echelon distribution network must be designed to last for several years, and robust enough to cope with all the random environmental factors (demand, prices, etc.) affecting the normal operations of a company. In our case, we focus on uncertain customer demands. Our planning horizon covers a set of periods (typically years) consolidated into cycles, and each cycle includes a set of operational periods. Hereafter, we reduce operational period to a typical day. Design decisions are revised at the beginning of cycles. Once the design decisions have been implemented, they are used on a daily basis to perform warehousing, distribution and transportation.

Under demand uncertainty, the design decision and the transportation decision give rise to a complex multi-stage decision problem. Using a planning horizon covering several cycles, the first-period design decisions are fixed but subsequent design decisions are re-considered as recourse to adapt the network to its environment. Additional information will be available at the beginning of each cycle, and it should be taken into account to obtain optimal redesign decisions. This leads to the solution of multi-period distribution design models on a rolling horizon basis [2]. Thus, the problem is modeled as a multi-cycle two-stage stochastic program with recourse. In this study, we suppose that the planning horizon consists in multi-year planning horizon where each year  $t$  (for instance cycle) is characterized by its own set of demand scenarios  $\Omega_t$  modeling the uncertainty behavior of demands in different zones. Each scenario

represents a typical day of delivery. An outsourcing strategy is allowed to satisfy demands with a higher cost if it cannot be delivered through the designed network.

## 2 Tackling the problem

We are developing approach to solve the problem for large scale instances, comparing solving directly the deterministic equivalent model versus applying Benders decomposition. Our numerical test are based on a sample average approximation.

A typical problem instance in a practical case would have an infinite number of scenarios and the probabilities cannot be estimated. In order to obtain solvable stochastic program, one needs to reduce the size of the problem using a smaller set of scenarios. This leads to use a sample average approximation (SAA) [3, 4] which involves solving the two-stage problem with a much smaller number of scenarios than the original problem. In fact, an independent sample of  $n$  equiprobable scenarios  $\Omega_t^n \subset \Omega_t$  is generated from the probability distribution using Monte Carlo procedure that approximates the expected recourse costs. Then, the SAA program is obtained. The quality of the solution obtained with this approach improves as the size  $n$  of the sample of scenarios used increases.

However, one would choose  $n$  taking into account the trade-off between the quality of the solution obtained for the SAA problem and the computational effort needed to solve it. So, solving the SAA problem with independent samples repeatedly can be more efficient and will involve statistical evaluation on the quality of the approximate solutions.

## 3 Further work

A Monte Carlo procedure and a SAA model are under implementation to tackle the multi-cycle stochastic two-echelon distribution design network. Computational experiments on several instances are under development to validate the approach.

## Références

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