Mobile Guards to Oversee an Art Gallery

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1 Introduction

Suppose that an art gallery is represented by a polygon. The objective of the Art Gallery Problem (AGP) consists in finding the minimum number of guards that we need to oversee the whole polygon [4, 6, 7]. The AGP is a well-known NP-hard problem in Computational Geometry and has applications in various domains e.g., telecommunications and robotics. There are several variants of the AGP that are based on different assumptions or applications [1, 6, 7]. In this study, we are interested in a variant of AGP where the guards are mobile, autonomous, and have limited communication abilities. In absence of full knowledge about the polygon, the objective is no more finding the minimum number of guards, but rather, we look for solving a mix of decision as well as navigation problems. More precisely, throughout navigating (properly) inside a given polygon, the guards need to decide where should they place themselves. However, we still expect a full coverage of the given polygon. For solving this variant of AGP, we present a self-contained platform to simulate the problem and the solution methods [5]. Furthermore, we introduce a new algorithm and evaluate its efficiency in comparison to classical methods. For this purpose, we use standard instances. The numerical results confirm that our algorithm outperforms the classical methods.

2 Formal Description of the Problem

2.1 Notation and Basic Definitions

Let Q be a non-convex simple polygon that contains no holes and represents an art gallery. Suppose that $Ve(Q) = \{v_1, \ldots, v_n\}$ and $E(Q) = \{e_1, \ldots, e_n\}$ are, respectively, the vertex set and the edge set of Q. Such a polygon can be partitioned into m polygons Q_i, where $i = 1, \ldots, m$ [2, 3, 5]. We denote by $P(Q)$ all points of Q that lie inside or on the edges of Q. A *diagonal* of Q is a line connecting two non-consecutive vertices $v_i, v_j \in Ve(Q)$, such that $]v_i, v_j[\in int(Q)$, where $int(Q)$ is the set of interior points of Q. A diagonal of Q is called a *gap* if and only if there exist two adjacent partitions that share this diagonal [2, 3]. For any two points $p, q \in P(Q)$, q is *visible* from p if and only if $[p, q] \in P(Q)$. For any given point $p \in Q$, all points $q \in Q$ that are visible from p define the *visibility polygon* $S(p) \subset Q$ of p. A polygon Q_{star} is called *star-shaped* if and only if it contains at least one point from which all point of Q_{star} are visible. Similarly, we define the *vertex-limited* visibility polygon $S_{ver}(p) \subset Q$ as the polygon formed by the vertices of Q that are visible from point p. A *kernel point* is defined as a point from which all points of a polygon is visible [7].

2.2 Art Gallery Deployment Problem

Suppose that a set A of (autonomous mobile) guards is given. The term autonomous means that the guards can sense their environment and can communicate with the guards that are located within a certain distance. The objective of the Art Gallery Deployment Problem (AGDP) consists in finding some points of a given polygon Q to place guards in order to oversee the whole polygon. This is slightly different from the classical Art Gallery Problem in which we are looking for the minimum number of guards ; however, in the distributed version, we assume that the guards have no complete knowledge about the polygon, prior to the deployment [2]. In this study, we restrict ourselves to vertex guards, i.e., the guards can only be placed on vertices of the polygon.

3 Solution Methods

Let us suppose that a polygon Q and a set A of n guards $a_1, \ldots, a_n$ are given. First, we assume that each guard has a unique identifier (id) and is equipped with an omnidirectional line-of-sight sensor of unlimited range [2, 3]. Furthermore, each guard has a limited memory M_i for saving necessary information. We update M_i a soon as the guard a_i moves or if it receives some information from other nearby guards. The outline of the procedures for solving AGDP is as follows :

- Partitioning Q into a set of star-shaped polygons $\{Q_1, \ldots, Q_m\}$. For this purpose, starting from a vertex $s \in Ve(Q)$, we find the vertex-limited visibility polygon of s which determines the first partition. We repeat the procedure on the remaining parts of Q by moving from a partition, through a gap, to a vertex that is close to the current partition.
- Throughout partitioning, we collect in the set $K^*(Q)$, one kernel point of each partition.
- Starting the root node $s = k_1$, we construct the tree $T^*(Q)$ by using $K^*(Q)$. We can prove that if Q has N vertices, then $T^*(Q)$ has, at most, $\lfloor N/2 \rfloor$ nodes [3].
- Throughout the construction, we explore $T^*(Q)$ and place guards at its vertices. These guards oversee the whole polygon because they are at the kernel points of partitions.

Several approaches can be used for exploring $T^*(Q)$. The well-known Depth-First Search (DFS) as well as a random search have already been adapted to this purpose [2, 3, 5]. Our main contribution consists in introducing a novel approach that is, indeed, a smart combination of the aforementioned methods. In fact, the new algorithm uses advantages of each method that are systematic exploration of $T^*(Q)$ and diversification of the search.

In order to evaluate the efficiency of algorithms, we carried out some numerical experiments, under same conditions, by using our self-contained AGP simulator [5]. For this purpose, we used benchmark as well as randomly generated instances of different sizes. According to our observations, our algorithm outperforms, in terms of computation time, the other methods.

Références

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