Improved Branch-Cut-and-Price algorithm for the Heterogeneous Vehicle Routing Problem

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1 Introduction

In this problem, a fleet of heterogeneous vehicles of different capacity is available to serve a set of customers with different demands. Each vehicle has to start and end its tour at the same depot. The cost of a tour of a vehicle is the sum of the fixed cost of the vehicle and the total distance of this tour multiplied by the "weight" of the vehicle. The problem consists in finding a set of vehicle routes that serve all the customers such that the sum of the quantities delivered in each tour does not exceed the capacity of the vehicle used in the tour and the total cost is minimized. The problem has several different variants depending of whether 1) there are fixed vehicle costs or not, 2) there are different vehicle "weights" or they are the same, 3) there are upper bounds of the number of available vehicles of the same type of not.

To our knowledge, the best exact approaches for the problem are proposed in [5] and in [2]. The later considers five variants of the problem and presents computational results on the instances of Taillard [7], with up to 100 customers. All but five instances were solved to optimality. The best published heuristic for the problem was proposed in [6] and provides the current best known solutions for all Taillard instances and also for a set of Brandão instances [1] with up to 199 customers.

2 Branch-Cut-and-Price Algorithm

In the work, we present an extension of the branch-cut-and-price algorithm of [4] to the case of heterogeneous vehicles. It uses a state-of-the-art bi-directional labeling algorithm to solve the pricing problem which decomposes to a set of non-elementary resource-constrained shortest path problems, one for each vehicle type. The labeling algorithm supports ng-routes and limited memory rank-1 cuts, which are used to strengthen the relaxation of the problem. Additionally we use reduced cost fixing and enumeration of elementary routes techniques to speed-up pricing problem resolution. We branch first on the number of vehicles of a same type and then on the arcs of the graph of customers. Multi-phase strong-branching with pseudo-costs is used to improve the selection of branching candidates.

The original contributions of this work in comparison with [4] are : 1) the use of cuts over the extended variables introduced in [5], even with the rank-1 cuts they are shown to be still important for solving some instances ; 2) the concept of subproblem dependent memory for rank-1 cuts, that sharper memory definitions allows more cuts to be added without making the pricing too expensive ; 3) We propose a way of lifting the coefficients of columns-routes in rank-1 cuts that takes advantage of the fact that the enumeration of routes is subproblem dependent.

3 Computational Results

Our branch-cut-and-price algorithm was able to solve all Taillard instances considered in [2] to optimality. The maximum instance solution time is 1 hour and 35 minutes (all instances except one were solved in less than 30 minutes). Improved feasible solutions were found for two instances.

Our algorithm was able to solve seven out of ten Brandão instances to optimality. The largest solved instance has 199 customers and took 12 hours. The smallest unsolved instance has 120 customers. Improved feasible solutions were found for four instances.

We have also tested our algorithm with a time limit of 24 hours on 96 DLP instances [3] which have from 20 to 256 customers. Each instance is based on the map of one of departments of France. Our algorithm solved to optimality 14 (out of 15) instances up to 100 customers, 25 (out of 38) instances with 101-150 customers, 9 (out of 30) instances with 151-200 customers. Improved feasible solutions were found for 23 instances.

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