

“Stochastic Programming Approaches for Planning Re-manufacturing Activities
under Uncertain Returns and Demand Forecasts“

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1 Introduction

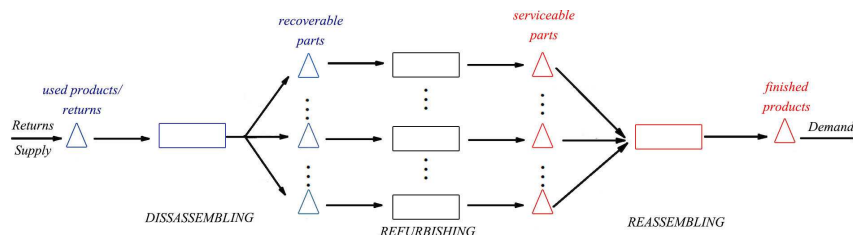
Due to the growing pressure to reduce the environmental impact of production processes while optimizing the economic profit, industrial companies around the world take increasing concern on the use of *reverse supply chains*. One of the activities found in reverse supply chains is *re-manufacturing*. It is defined as a set of processes transforming end-of-life products (used products/ returns) into like-new finished products, once again usable by customers, mainly by rehabilitating damaged components. In the present work, we consider a re-manufacturing system involving three production echelons: *Disassembling, Refurbishing, and Reassembling*. We focus on optimizing the production planning of this re-manufacturing system over a multi-period horizon, which leads to the formulation of a multi-echelon lot-sizing problem.

One of the main challenges when dealing with re-manufacturing is the high level of uncertainty in the input data such as the quantity of returns supply and the market demand forecasts, arising from the lack of control by companies on the quantity, quality and timing of the used products returned by customers.

The contributions of the present work are threefold. Firstly, we study a multi-echelon re-manufacturing system while considering the limited availability of returned products. This is in contrast with most previously published works which either consider a single-echelon system or assume an unlimited quantity of returns. Second, we propose a multi-stage stochastic programming approach capable of explicitly handling the uncertainty in the problem input data. This approach leads to the formulation of large-size Mixed-Integer Linear Programming problems. Finally, we develop a solution algorithm based on polyhedral approaches to solve the problem. Note that a similar approach was proposed in [Guan et al., 2009] but for a much simpler single-echelon single-item lot-sizing problem. Our preliminary computational results show that the proposed approach is efficient at solving the small to medium size instances.

2 Outline of The Presented Work

Over a discrete time horizon, we seek to build an optimal production plan for a re-manufacturing system involving three main production echelons: (1) *disassembling* the used product into a series of recoverable parts, (2) *refurbishing* the recoverable parts into serviceable parts on dedicated rehabilitating processes, (3) *reassembling* the serviceable parts into re-manufactured products (see the following illustrative figure).



Firstly, we consider a *Deterministic version* of the problem, where we temporarily assume that all input parameters are determined beforehand. We study a solution approach on this deterministic problem, we then observe

the behaviours of reformulation techniques and the computational performances of our specially designed Cut-and-Branch algorithm on solving it. Based on these results, we have some insights on how to solve the *Stochastic version* of the problem, where the returns quantities and the customers demand in each period will only be unfolded little by little as realizations of discrete-time stochastic processes.

Our main focus is solving this Stochastic problem to optimality, and we seek for an algorithm with sufficiently effective performance in solving up to medium-size practical instances. This has been done by the following steps:

- We represent the uncertain information structure by multi-stage stochastic programming approaches (allowing the possibility to re-adjust the production plan based on the information unfolded later on), explicitly by a scenario tree. Each node n on the tree corresponds to a single planning period t belonging to a single decision stage s , and n represents the state of the world that can be distinguished by the information unfolded up to that period t .
- This leads to the first “Natural Formulation” (STO-NF) of the considered problem as a MILP (which has similar structure to the investigated deterministic problem). Its objective function aims at minimizing the total expected cost, over all nodes of the scenario tree.
- We then reformulate (STO-NF) into the “*Stochastic Echelon Stock Formulation*” (STO-EF) by using the echelon stock concept. This new reformulation circumvents the difficulties encountered with (STO-NF) and allows us to consider (STO-EF) as a series of simpler sub-problems linked together by some coupling constraints.
- We investigate these sub-problems, find the correspondingly modified form of the (k, U) valid inequalities (proposed in [Loparic et al., 2001]). We then write a separation algorithm with these (k, U) cuts on each of the sub-problems.
- By computational experiments, we calibrate and find a well-performing strategy in applying these separation algorithms on our specific scenario tree structure. We then design a Cut-and-Branch algorithm for solving the whole (STO-EF) problem. We set up some computational experiments to test our Cut-and-Branch algorithm and observe the results.
- We suggest a new perspective attempting to create new and theoretically stronger valid inequalities for future research.

3 Conclusion and Perspectives

We have found an algorithm succeeding in solving to optimality our considered stochastic lot-sizing for re-manufacturing problem (which is a NP-hard problem), with well-performing computational results on medium-size instances. This can be considered as the first step toward a potential algorithm capable of solving real-life practical problems, as well as a benchmark for comparing and calibrating the heuristics approaches. The results obtained from our project also give theoretical knowledge in the approaches of strengthening lot-sizing problems by echelon stock reformulation and adding (k, U) cuts.

References

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