A joint chance-constrained programming approach for the single-item capacitated lot-sizing problem with stochastic demand

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1 Introduction

We study a combinatorial optimization problem arising in the context of production planning, namely the single-item single-resource capacitated lot-sizing problem. Most often, one of the key input data of this optimization problem, the future customer demand to be satisfied, is not perfectly known at the time when the production plan is built, resulting in a stochastic combinatorial optimization problem. In the present work, we assume that, even if the demand cannot be deterministically known, a description of the demand uncertainty is available in terms of a probability distribution. We propose to handle the problem through the use of a single-stage stochastic programming approach in which the whole production plan is built before any additional information on the demand realization becomes available.

2 Problem formulation

We wish to plan production for a single product to be processed on a single capacitated resource over a planning horizon involving T periods indexed by $t = 1...T$. The cost and capacity parameters are assumed to be deterministically known : f_t denotes the fixed setup cost to be paid if production occurs on the resource in period t, h_t the inventory holding cost per unit held in stock at the end of period t and c_t the production capacity available in period t. The initial inventory level, s_0, is set to 0 without loss of generality.

We model the demand in period t as a random variable D_t, the probability distribution of which is assumed to be known. The fact that D_1, D_2, ..., D_T are stochastic implies that it might be very costly (and even impossible) to build a production plan ensuring that the cumulated production over $1..t$ is large enough to satisfy every possible realization of the cumulated demand over $1..t$. We thus have to consider the eventuality that inventory shortages will occur for some demand realizations. A possible way of handling this situation in the problem modeling consists in limiting the probability of these inventory shortages through the use of chance constraints. We thus define a maximum acceptable risk level ϵ and impose that the probability that the production plan satisfies the demand in all periods of the planning horizon is above $1 - \epsilon$.

We introduce the following decision variables :

— x_t : the quantity produced in period t.

— $y_t \in \{0, 1\}$: the resource setup state in period t. $y_t = 1$ if a setup occurs in period t, 0 otherwise.

This leads to the following chance-constrained programming formulation.

$$\begin{cases} Z = \min \sum_{t=1}^{T} f_t y_t + \sum_{t=1}^{T} h_t max(\sum_{\tau=1}^{t} x_\tau - \sum_{\tau=1}^{t} D_\tau, 0)\Big) & & (1) \\ x_t \leq c_t y_t & \forall t = 1...T & (2) \\ \Pr\Big(\sum_{\tau=1}^{t} x_\tau \geq \sum_{\tau=1}^{t} D_\tau, \forall t = 1...T\Big) \geq 1 - \epsilon & & (3) \\ x_t \geq 0 & \forall t = 1...T. & (4) \\ y_t \in \{0, 1\} & \forall t = 1...T. & (5) \end{cases}$$

The objective function (1) corresponds to the minimization of the setup costs and of the expected inventory holding costs over the planning horizon. Note that $\sum_{\tau=1}^{t} x_\tau - \sum_{\tau=1}^{t} D_\tau$ computes the inventory level at the end of period t as the difference between the cumulated production up to t and the cumulated demand up to t. Constraints (2) ensure that, if production takes place in period t, the corresponding setup costs are incurred and the capacity limit c_t is respected. Constraints (3) is a joint chance constraint ensuring that the probability that all demand satisfaction constraints are simultaneously satisfied by the production plan, i.e. that no inventory shortage occur over the whole planning horizon, is above $1 - \epsilon$.

3 Solution approach

As the resulting probabilistic mixed-integer program is computationally difficult to handle, we consider an approximate solution approach based on the sample approximation approach discussed in [1]. The main idea of this approach consists in replacing the original continuous probability distribution of the random variables (D_1, D_2, ..., D_T in our case) by an empirical discrete finite probability distribution obtained by Monte Carlo sampling. This approach leads to the formulation of a mixed-integer linear program. However, due to the large number of additional binary variables introduced in the problem modeling, the sample approximation approach leads to a significant computation time when the Monte Carlo sample size increases, even when advanced mixed-integer linear programming techniques (e.g. extended reformulation) are used.

This is why we propose a new extension of this approach for the case where the demand in the first period D_1 is assumed to be statistically independent of the demand in the other periods and to follow a continuous uniform distribution. Similarly to the the sample approximation approach, the proposed extension relies on a Monte Carlo sampling method. However, the sampling is not carried out on all the random variables involved in the stochastic problem (i.e. on D_1, D_2, ..., D_T in our case) but only on part of them (more precisely on D_2, ..., D_T in our case). We thus refer to it as the partial sample approximation approach. Its main advantage is that, contrary to the sample approximation approach, it leads to the formulation of a deterministic mixed-integer linear problem having the same number of binary variables as the original stochastic problem.

Our computational results carried out on instances involving up to 20 periods and 5000 scenarios in the sample size show that the proposed solution method consistently provides feasible or near-feasible solutions of the original stochastic problem within significantly reduced computation times as compared to the sample approximation method.

Références

[1] Luedtke J., Ahmed S. and Nemhauser G.L., An integer programming approach for linear programs with probabilistic constraints, *Mathematical Programming*, 122, 247-272, 2010.