Reliable Hub Location Problem under Uncertainty

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***Mots-clés* : Hub location problem; Uncertainty; Disruption; Congestion; Multi-objective optimization.**

1 Introduction

The uncertainties in hub location problems (HLPs) can be generally categorized into hub-side uncertainty, spoke-side uncertainty, and connection-link uncertainty [1,2]. The hub uncertainty are often caused by the randomness in hub capacity and the reliability of hubs, etc.; the spoke uncertainty are often due to the randomness in flows; and the link uncertainty could be due to the random travel time, random transportation cost, unreliable routes, etc. Most existing literature on HLPs under uncertainty focus on a limited number of spoke and link uncertainties such as uncertain flows, transportation costs and times (see [3] and the references therein) and they have ignored the hubs and links uncertainties that highly impact the functional performance of the hub network.

Another unrealistic assumption is that the located hubs and established links are always operational as planned, and the topology of the hub-and-spoke network is stable. However, in practice, hubs and links could fail due to different factors (e.g., adverse weather affects the availability of the airports and links. Therefore, disrupted hubs should be temporarily or permanently relocated and the disrupted links should be re-routed.

Two complete and partial disruptions can happen in the hubs and links. In complete disruption, hubs and links become completely unavailable. In partial disruption, both hubs and links are still working while their capacity is degraded to a lower level.

To the best of our knowledge, there exist few papers dealing with disruption in HLPs. Zeng et al. [4] designed reliable hub-and-spoke networks by taking hub unavailability into account with given reliability value for each hub. They observed that in comparison with the reliable facility location models, reliable hub-and-spoke models are much more complicated. Parvaresh et al. [5,6] formulated a bi-level multiple allocation *p*-hub median problem under intentional disruptions by a bi-level model with bi-objective functions at an upper level and a single objective at a lower level. Mohammadi et al. [1] proposed a single-objective mixed-integer programming (MIP) model for a reliable logistics network through a hub location network that is less sensitive to disruptions only in the hubs. The literature did not consider complete and partial disruptions in the hubs and the links as well as the effect of these uncertainties on the network functionality performance.

In this paper, we propose a new bi-objective mixed integer non-linear programming (MINLP) model for the HLP by taking into account the complete and partial disruptions. A new meta-heuristic algorithm is also developed and applied on the real data to obtain the Pareto solutions.

2 Mathematical formulation

This section provides the proposed mathematical model. Due to space limitation, most of the explanation has been ignored; but they can be provided upon request. The notations are first defined as follows.

Sets:: i, $j \in N$: set of nodes; i, $j \in H$, $H \in N$: set of hubs; $m \in M$: set of transportation modes; $s \in S$: set of capacity levels; $r, v \in R$: set of allocation levels. **Parameters**:: w_{ij}: flow between spokes i and j; c_{ij}^{m}:

transportation cost of a unit of flow between spokes *i* and *j* using transportation mode m; oc_k^{ms}: unit operational cost at hub *k* with capacity level *s* using transportation mode *m*; FH_k^{ms}: fixed cost of locating a hub at node *k* with capacity level *s* using transportation mode *m*; FL_{kl}^{m}: fixed cost of link between hubs *k* and *l* using transportation mode *m*; αc_{ij}^{m}: cost discount factor between hubs *k* and *l* using transportation mode *m*; αt_{ij}^{m}: time discount factor between hubs *k* and *l* using transportation mode *m*; $\bar{\Gamma}_k^{sm}$: designed capacity of hub *k* with capacity level *s* using transportation mode *m*; $\bar{\xi}_{kl}^{m}$: designed capacity of the link between hubs *k* and *l* using transportation mode *m*; q_k: failure probability of complete disruption of hub *k*; *P*: number of hubs to be located; $O_i = \sum_j w_{ij}$: total flow originating from spoke *i*; $D_i = \sum_j w_{ji}$: total flow with destination of spoke *i*; μ_k^{ms}: service rate of hub *k* with capacity level *s* using transportation mode *m*; f_k^{ms}: disruption rate of hub *k* with capacity level *s* using transportation mode *m*; r_k^{ms}: retrieval time rate of hub *k* with capacity level *s* using transportation mode *m*; η_k^{ms}: disruption probability at hub *k* with capacity level *s* using transportation mode *m*; θ_k^{ms}: capacity disruption factor at hub *k* with capacity level *s* using transportation mode *m*; ϑ_{kl}^{m}: disruption probability at link between hubs *k* and *l* using transportation mode *m*; δ_{kl}^{m}: capacity disruption factor at link between *k* and *l* using transportation mode *m*; t_{ij}^{m}: free-flow travel time between nodes *i* and *j* (spokes/hubs) using transportation mode *m* (t_{ij}^{m} is a deterministic parameter). **Variables**:: X_{ik}^{r}: 1 if spoke *i* is allocated to hub *k* at level *r*; Z_k^{ms}: 1 if a hub is established at node *k* with capacity level *s* using transportation mode *m*; Y_{iklj}^{m}: 1 if the flow originated at spoke *i* destined to spoke *j* uses the hub link {*k*,*l*} from hub *k* to hub *l* with transportation mode *m*; L_{kl}^{m}: 1 if there is a link between hubs *k* and *l* with transportation mode *m*; U_k: 1 if a hub is located at node *k*; P_{ik}^{r}: probability that hub *k* serves spoke *i* at level *r*; $EF_k^m = \lambda_k^m = \sum_i \sum_{j \neq i} \sum_{l \neq k} w_{ij} Y_{ilkj}^{m}$: flow entering the hub *k* using transportation mode *m*; $LF_{kl}^{m} = \sum_i \sum_j w_{ij} Y_{iklj}^{m}$: flow passing the link between hubs *k* to *l* using transportation mode *m*; $E(W_k^{ms})$: mean expected value of the stochastic operational time (waiting time + processing time) at hub *k* with capacity level *s* for transportation mode *m*; $V(W_k^{ms})$: variance value of the stochastic operational time (waiting time + processing time) at hub *k* with capacity level *s* for transportation mode *m*; W_k^{ms}: operational time (waiting time + processing time) at hub *k* with capacity level *s* for transportation mode *m* ($W_k^{ms} \sim (E(W_k^{ms}), V(W_k^{ms}))$); $E(T_{ij}^{m})$: mean expected value of the stochastic travel time between nodes *i* and *j* using transportation mode *m*; $V(T_{ij}^{m})$: variance value of the stochastic travel time between nodes *i* and *j* using transportation mode *m*; T_{ij}^{m}: transportation time between nodes *i* and *j* using transportation mode *m* ($T_{ij}^{m} \sim (E(t_{ij}^{m}), V(t_{ij}^{m}))$).

The proposed mathematical model is as follow:

$$\min \text{Obj}_1 = \sum_{k=1}^{N} \sum_{m=1}^{M} \sum_{s=1}^{L} FH_k^{ms} Z_k^{ms} + \sum_{k=1}^{N+1} \sum_{m=1}^{M} \sum_{\substack{l=1 \\ l>k}}^{N+1} FL_{kl}^{m} L_{kl}^{m} + \sum_{s=1}^{L} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_{ij} \Bigg[\sum_{\substack{k=1 \\ k \neq i}}^{N+1} \sum_{r=1}^{R} c_{ik}^{1} PX_{ik}^{r} + \sum_{m=1}^{M} \sum_{k=1}^{N+1} \sum_{\substack{l=1 \\ l \neq k}}^{N+1} \sum_{r=1}^{R} \sum_{r'=1}^{R} (oc_k^{ms} + \alpha c_{kl}^{m} c_{kl}^{m} + oc_l^{ms}) P_{ik}^{r} P_{jl}^{r'} Y_{iklj}^{m} + \sum_{\substack{k=1 \\ k \neq j}}^{N+1} \sum_{r=1}^{R} c_{kj}^{1} PX_{jk}^{r} \Bigg] \quad (1)$$

$$Min\ \text{Obj}_2 = \Psi \quad (2)$$

s.t.

$$\sum_{k=1}^{H} X_{ik}^{r} + \sum_{v=1}^{r} X_{i(H+1)}^{v} + U_i = 1 \qquad \forall i, r \quad (3)$$

$$\sum_{r=1}^{R} X_{i(H+1)}^{r} + U_i = 1 \qquad \forall i \quad (4)$$

$$X_{ik}^{r} \le U_k \qquad \forall i, k \quad (5)$$

$$\sum_s Z_k^{ms} \le U_k \qquad \forall k, m \quad (6)$$

$$\sum_s Z_k^{ms} = 1 \qquad \forall k, m \quad (7)$$

$$\sum_k U_k = P \quad (8)$$

$$L_{kl}^{m} \le \sum_s Z_k^{ms} \qquad \forall k, l: k < l, m \in M \backslash \{1\} \quad (9)$$

$$L_{kl}^{m} \le \sum_s Z_l^{ms} \qquad \forall k, l: k < l, m \in M \backslash \{1\} \quad (10)$$

$$L_{kl}^{1} \le \sum_m \sum_s Z_k^{ms} \qquad \forall k, l: k < l \quad (11)$$

$$L_{kl}^{1} \le \sum_m \sum_s Z_l^{ms} \qquad \forall k, l: k < l \quad (12)$$

$$\sum_m Y_{iklj}^{m} \ge X_{ik}^{r} + X_{jl}^{v} - 1 \qquad \forall i, j, k, l: i \neq j; k \neq l; r, v \quad (13)$$

$$Y_{iklj}^{m} + Y_{ilkj}^{m} \le L_{kl}^{m} \qquad \forall i, j, k, l \in N: i \neq j \quad (14)$$

$$P\left\{ \left(T_{ik}^{1} + W_k^{ms} + \alpha t_{kl}^{m} T_{kl}^{m} + W_l^{ms} + T_{lj}^{1} \right) Y_{iklj}^{m} \le \Psi \right\} \ge \gamma \qquad \forall i, j, k, l, m, s: i \neq j \quad (15)$$

$$\sum_i \sum_l \sum_j \sum_r w_{ij} PY_{ilkj}^{mr} \le \sum_s \bar{\Gamma}_k^{ms} [\theta_k^{sm} + (1 - \eta_k^{ms})(1 - \theta_k^{sm})] Z_k^{ms} \qquad \forall k, m \quad (16)$$

$$\sum_i \sum_j \sum_r \sum_v w_{ij} Y_{iklj}^{m} P_{ik}^{r} P_{jl}^{v} \le \bar{\xi}_{kl}^{m} [\delta_{kl}^{m} + (1 - \vartheta_{kl}^{m})(1 - \delta_{kl}^{m})] L_{kl}^{m} \qquad \forall k, l, m \quad (17)$$

$$P_{ik}^{1} = 1 - q_k \qquad \forall i, k \in \{1, \dots, H+1\} \quad (18)$$

$$P_{il}^{r} = (1 - q_l) \sum_{k=1}^{H} \frac{q_k}{1 - q_k} PX_{ik}^{r-1} \qquad \forall i, l \in \{1, \dots, H+1\}, r \in \{2, \dots, R\} \quad (19)$$

$$PX_{ik}^{r} \le P_{ik}^{r}, PX_{ik}^{r} \le X_{ik}^{r}, X_{ik}^{r} + P_{ik}^{r} - PX_{ik}^{r} \le 1 \qquad \forall i;\ k \in \{1, \dots, H+1\}; r \quad (20)$$

$$PY_{ilkj}^{mr} \le P_{jk}^{r}, PY_{ilkj}^{mr} \le Y_{ilkj}^{m}, Y_{ilkj}^{m} + P_{jk}^{r} - PY_{ilkj}^{mr} \le 1 \qquad \forall i, j, k, l, m, r \quad (21)$$

$$X_{ik}^{r}, Z_k^{ms}, L_{kl}^{m}, U_k, Y_{ilkj}^{m} \in \{0,1\}; PX_{ik}^{r}, W_k^{ms}, T_{kl}^{m}, \Psi \ge 0 \qquad \forall i, j, k, l, m, s, r \quad (22)$$

Objective function (1) minimizes total expected transportation and operation cost in the hub network. Objective function (2) and chance constraint (15) minimize the maximum transportation time between each pair of O-D nodes. Equation (3) enforces that for each spoke i and each level r, either i is allocated to a regular hub at level r or is allocated to the non-failable hub $H+1$ at certain level $v \leq r$ (taking $\sum_{v=1}^{r} X_{i,H+1}^{v} = 0$ if $r = 1$). Constraint (4) requires each spoke to be allocated to the non-failable hub at a certain level. Constraint (5) ensures that a spoke must be allocated to a valid hub. Constraints (6) and (7) guarantee that just one capacity level is allowed for each located hub. Constraint (8) ensures the number of hubs should be equal to a pre-defined value P. Constraints (9) and (10) show that specific capacity levels in both hubs k and l for mode m must be established if there is a link between them with mode m. Constraints (11) and (12) explain that any hub constructed for mode $m \geq 2$ can be utilized also for mode m equal to 1. In this model, mode equal to 1 is considered as the road transportation. Constraints (13) and (14) create valid route between each pair of O-D nodes. Constraints (16) and (17) are the hub and the link capacity constraints, respectively. Constraints (18) and (19) are the "transitional probability" equations. P_{ik}^{r}, the probability that hub k serves spoke i at level r, is just the probability that k remains open if $r = 1$. For $2 \leq r \leq R$, P_{ik}^{r} is equal to $(q_l\,(1-q_k)/(1-q_l))P_{il}^{r-1}$ given that hub l serves spoke i at level r. Constraints (20) and (21) make the terms of $X_{ik}^{r} \times P_{ik}^{r}$ and $Y_{ilkj}^{m} \times P_{jk}^{r}$ linear, respectively. Finally, constraints (22) are domain constraints.

3 Solution approach

In this section, a novel solution approach is developed to solve the proposed model. This approach consists of: 1) exact approximation of the proposed model, 2) multi-objective lower bound (MOLB) procedure, and 3) multi-objective meta-heuristic algorithm. For the exact approximation, the interested readers are referred to [1]. For the multi-objective lower bound, the proposed augmented *e*-constraint method of Mavrotas [7] is combined by the lower bound approach proposed by [8], wherein by varying the *e* value, non-dominated lower bound solutions are extracted.

We proposed a hybrid meta-heuristic algorithm based on new self-adaptive non-dominated genetic algorithm II (SNSGA-II) and variable neighborhood search (VNS) algorithm, namely SGV-II, in order to find non-dominated front (NF) solutions near to optimal Pareto frontier (PF) of the proposed model. In the SNSGA-II, a self-adaptive version of crossover and mutation operators is applied, wherein different operators adapt themselves with solution space and more powerful operators have higher chance to guide the search algorithm. Once the SNSGA-II finds the initial set of non-dominated solutions, the VNS algorithm tries to locally improve the NF solutions.

4 Computational results

The results are threefold: (1) investigating the tightness of the proposed MOLB, (2) investigating the quality of the SGV-II, and (3) applying the proposed model and the solution approaches on real case. Figure 1 shows the tightness of the MOLB approach comparing to the optimal PF obtained by the Cplex solver. The mean gap between the NF of MOLB and optimal PF was obtained equal to 1.05%. Figure 2 shows the outperformance of the SGV-II comparing to other well-known algorithms and the MOLB approach as well. After numerous experiments, the mean gap between the NF of SGV-II and optimal PF obtained equal to 0.98%.

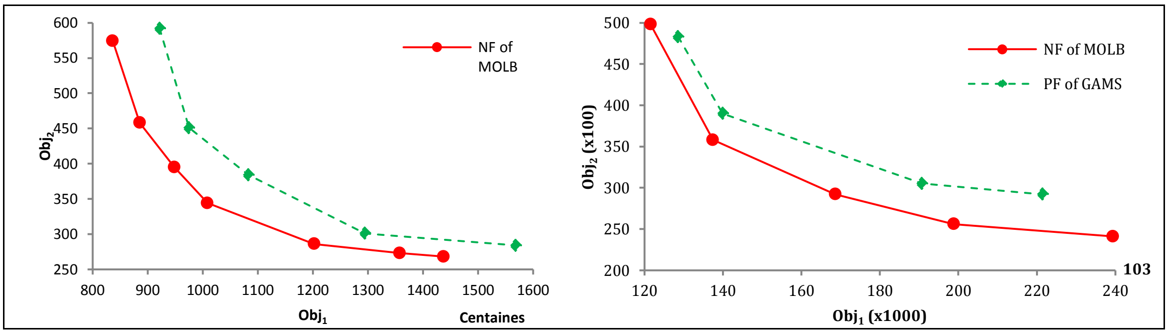

FIG 1. Optimal PF by CPlex and NF by MOLB on small-size (left) and medium-size (right) problems

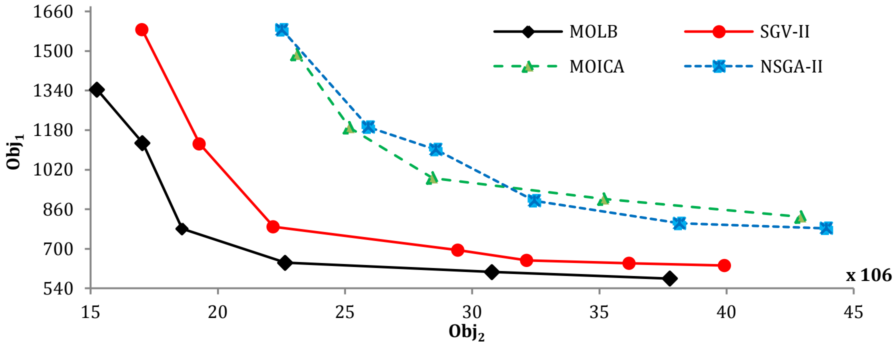

FIG 2. NFs of different meta-heuristics for a large-size problem

For the real case study, different scenarios with particular number of hubs have been considered and the results have been shown in Figure 3. It is clear that how the proposed model in this paper can provide solutions with lower values of the object functions in comparison with the current transportation network in Iran.

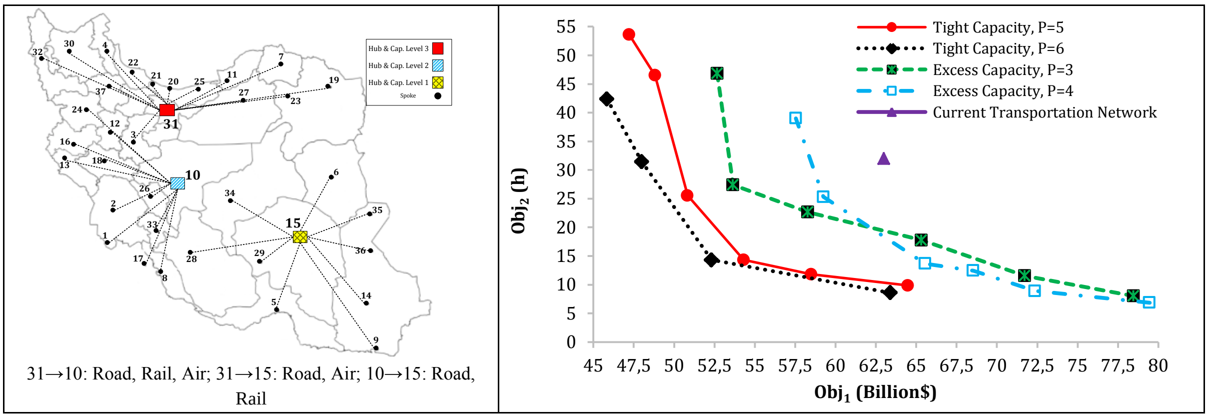

FIG 3. Comparison between the NF solutions of the proposed model with the current transportation network

5 Conclusion

In this paper, we focused on the reliability issues of hub uncertainties (complete and partial disruptions) and link uncertainties (partial disruptions). We proposed a bi-objective mathematical model and a new solution approach and we applied them on the Iranian transportation network.

To the best of our knowledge, our paper is the first to present a bi-objective reliable CHLP model with hub and link uncertainties. This paper also targets a gap in existing literature by providing an approximation algorithm and a lower bound approach for the bi-objective mathematical model. It is our hope that this study could inspire additional in-depth research and discussions on this topic.

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