Lagrangian relaxation approach for a multi-period network design and routing problem

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In today's communication networks, distributed control functions such as routing, are driven by path quality properties (such as cost and bandwidth delay product) but also their adaptation cost and convergence time. However, the design of the routing function (and associated routing protocol procedures) remains driven by their consumption of processing capacity and memory available locally at each node. Henceforth, the routing decision process (distributed and online) remains also decoupled from the network design process, i.e., resource installation and allocation planning process (centralized and offline).

The conventional model to formulate such problem assumes that routing decisions can be performed without informing the capacity optimization problem (link resource installation and (modular) allocation). These decisions are modeled in terms of capacity allocation per link but without accounting for the routing state creation and maintenance cost. This formulation is thus often extended by assuming that the routing optimization process can additionally inform the capacity installation and allocation process about its utility. The latter then adjusts the allocated capacity on each link and may decide to add new links (between node pairs not previously connected). This method has been applied for instance to various combined network design and traffic engineering problems including IP over Multi-Protocol Label Switching - Traffic Engineering (MPLS-TE) and IP over optical/wavelength switching layer. However, such formulation does not account for i) the cost associated to the creation of a routing adjacency once the corresponding link is added, ii) the cost of link maintenance during the lifetime of the corresponding routing adjacency, and iii) the routing cost function which remains independent of the link occupancy.

For these reasons, we study an extension of the multi-commodity capacitated fixed charge network design (MCND) problem introduced by [3, 4, 6] which deals with the simultaneous optimization of capacity installation cost and traffic flow routing cost. In the MCND problem, a fixed cost is incurred for opening a link and a linear routing cost is paid for sending traffic flow over an edge (or arc). The routing decision must be performed such that traffic flows remain bounded by the installed capacities. In [7], we generalized this problem over multiple time periods using an increasing convex routing cost function which takes into account congestion (number of routing paths per edge) and delay (routing path length). A compact Mixed Integer Linear Program (MILP) formulation for this problem is developed based on the aggregation of traffic flows following the per destination routing decision process underlying packet networks. However, the resolution with realistic topologies and traffic demands becomes rapidly intractable with state-of-the-art solvers due to the weak linear programming bound of the proposed MILP formulation. An extended formulation where traffic flows are disaggregated by source-destination pairs, while keeping the requirement of destination-based routing decisions has been studied in [8].

In general, direct formulations for determining optimal routing decisions obeying various protocol rules are complex to solve. By relaxing the linking constraints, the Lagrangian relaxation method can be applied to the base (aggregated) and extended (disaggregated) formulation in order to provide stronger lower bounds. Moreover, the suitable choice of the complicating constraints yields a Lagrangian subproblem decomposable by node, in line with the objective of obtaining a decomposition of the original optimization problem which preserves the distributed nature of the routing decision process.

Multi-commodity fixed charge network design problems are extremely challenging to solve. This complexity arises because even the simple continuous versions usually contain a huge number of variables and constraints, which makes them very hard to solve with standard approaches. Indeed, specialized algorithms are required [2, 5] and the use of parallel architectures could be necessary [1]. The complexity becomes even higher if integer variables are present in the models to represent logical decisions. The resulting mathematical model is a MILP with multi-commodity network flow structure.

The model considered in this talk extends the MCND problem by including different types of fixed costs (installation and maintenance costs) and variable costs (routing costs). In addition, time dependent demands are taken into account and the network is designed for more than one time period. We propose a Lagrangian relaxation approach for computing a lower bound, and a Lagrangian heuristic for obtaining good feasible solutions. For this purpose, we relax the flow conservation equations such that the Lagrangian subproblem can be decomposed by node. We remark that compared to what happens in the standard Fixed Charge Network Design problem, the Lagrangian subproblem is not a knapsack problem. Unfortunately, this yields a Lagrangian subproblem that is not so easy to solve. However, the approach is more robust than using CPLEX as a black-box MIP solver, as the Lagrangian relaxation always returns a lower bound and provides a feasible solution for many instances where CPLEX fails.

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