Scheduling with sequence-dependent setup times and periodic maintenance on a single machine to minimize the total weighted completion time.

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1 Introduction

Most of the scheduling problems consider that the machines are always available and in optimal conditions. But in real production situations, this assumption may not be valid due to maintenance or other unexpected events. An efficient planning of periodic maintenance can improve production efficiency and safety. Therefore, scheduling the maintenance has gradually become a common practice in many companies. In a previous work [5] we proposed a mixed integer linear program formulation and several heuristics to solve a single machine scheduling problem with periodic maintenance so as to minimize the weighted sum of completion time. As an extension of that problem, we consider in the present work to add the constraint of sequence dependent-setup times which makes this scheduling problem more realistic. Sequence dependent means that each setup time depends on the job that is processed immediately before. This is frequently observed in real industry settings as in the textile, manufacturing printed circuit boards and chemical industries [4, 2].

In this work, the objective is to compute a schedule that minimizes the weighted completion times, subject to sequence-dependent setup time and periodic maintenance. Referring to the notation of Graham et al. [3], we denote this scheduling problem as $1/pm, ST_{sd}/\sum_{i=1}^{n} w_i c_i$. To the best of our knowledge, only few works report studies on scheduling problems considering both sequence-dependent setup times and preventive maintenance constraints. Angel-Bello et al. [7] have considered such a scheduling problem with objective of minimizing the makespan.

On the basis of the comprehensive survey on scheduling problem with setup times done by Allahverdi in [1] and, to the best of our knowledge, no one considered such a problem in the related literature with the objective of minimizing the job weighted sum completion time.

2 Problem description

The problem under consideration in this paper can be stated as follows. A set $N = \{1, 2, \ldots, n\}$ of n independent jobs are simultaneously available for processing at the beginning of the scheduling horizon. Each job, with a deterministic processing time, has to be processed without preemption on a single machine that can handle only one job at a time. The objective function considered here is the weighted sum completion time $\sum_{i=1}^{n} w_i\, c_i$, where w_i is a positive weight for job J_i. Furthermore, the machine has to undergo a preventive maintenance in each period of time T. The maintenance activity occurs during the processing interval T. We consider δ as the duration of the maintenance. p_i, s_i and c_i represent, respectively, the processing time of job J_i, its starting time and its completion time. $st_{i,j}$, $st_{(n+1),i}$ and st_i are respectively the setup time between the two jobs J_i and J_j, the setup time between the maintenance and the job J_i and the setup time before the maintenance related to the job J_i. The studied problem is NP-Hard since Kirlik et al. [6] proved that the special case of minimizing the sum completion time without maintenance is NP-hard.

3 MIP Formulation

The $1/pm,ST_{sd}/\sum_{i=1}^{n} w_i c_i$ problem can be formulated as a mixed integer linear programming model by defining the following decision variables and considering the notation given earlier.

$$x_{ik} = \begin{cases} 1 & \text{if job } i \text{ is scheduled before job } k \\ 0 & \text{otherwise} \end{cases} \qquad y_{ij} = \begin{cases} 1 & \text{if job } i \text{ is in batch } j \\ 0 & \text{otherwise} \end{cases}$$

The set of jobs scheduled between two periodic maintenances defines a batch. We can have at most n batches. We define also R as a big positive integer.

$$min \quad \sum_{i=1}^{n} w_i c_i \tag{1}$$

$$\sum_{j=1}^{n} y_{ij} = 1 \qquad \forall i \in N, \tag{2}$$

$$c_k \geq c_i + st_{ik} + p_i - R(1 - x_{ik}) \quad \forall (i,k) \in N^2, i \neq j, \tag{3}$$

$$c_k \geq \delta + st_{(n+1),i} + p_i \quad \forall i \in N, \tag{4}$$

$$c_i \geq (j-1)Ty_{ij} + \delta + st_{(n+1),i} + p_i \quad \forall j \in 2..n, i \in N, \tag{5}$$

$$c_i \leq jTy_{ij} - \delta - st_i + R(1 - y_{ij}) \quad \forall (i,j) \in N^2, \tag{6}$$

$$x_{ik} + x_{ki} = 1 \quad \forall i \in N, k \in i+1..n, \tag{7}$$

$$c_i \geq \delta + st_{(n+1),i} + p_i \quad \forall i \in N, \tag{8}$$

$$x_{ik} \in \{0,1\} \quad \forall (i,k) \in N^2, \tag{9}$$

$$y_{ij} \in \{0,1\} \quad \forall (i,j) \in N^2. \tag{10}$$

4 Conclusion and perspectives

As stated in section 1, the $1/pm,ST_{sd}/\sum_{i=1}^{n} w_i c_i$ is NP-hard; hence, solving the above mathematical model to obtain an optimal solution is not practical (solving optimally problems with n=12 takes in average 45 minutes). Since we did not find test instances for the problem under study in the related literature, we decide to carry out experiments with a randomly generated instances. Our perspectives is to design new algorithms based on heuristics and metaheuristics (such as VNS) in order to find near-optimal solutions.

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